



Unit:1

Ratio, Proportion and Percentage:

Ratios And Proportion:

Ratio and Proportion are explained majorly based on fractions. When a fraction is represented in the form of a:b, then it is a ratio whereas a proportion states that two ratios are equal. Here, a and b are any two integers. The ratio and proportion are the two important concepts, and it is the foundation to understand the various concepts in mathematics as well as in science.

In our daily life, we use the concept of ratio and proportion such as in business while dealing with money or while cooking any dish, etc. Sometimes, students get confused with the concept of ratio and proportion

Examples

Word Problems

For example, $\frac{20}{25}$ is a ratio and the proportion statement is $\frac{20}{25} = \frac{4}{5}$. If we solve this proportional statement, we get:

$$\frac{20}{25} = \frac{4}{5}$$

$$20 \times 5 = 25 \times 4$$

$$100 = 100$$

Check: Ratio and Proportion PDF

Therefore, the ratio defines the relation between two quantities such as a:b, where b is not equal to 0.

Example: The ratio of 2 to 4 is represented as $2:4 = 1:2$. And the statement is said to proportion here. The application of proportion can be seen in direct proportion.

What is Ratio and Proportion in Maths?

The definition of ratio and proportion is described here in this section. Both concepts are an important part of Mathematics. In real life also, you may find a lot of examples such as the rate of speed (distance/time) or price (rupees/meter) of a material, etc, where the concept of the ratio is highlighted. Proportion is an equation which defines that the two given ratios are equivalent to each other. For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as $100\text{km/hr} = 500\text{km}/5\text{hrs}$.

Ratios and Proportion

Let us now learn Maths ratio and proportion concept one by one. Definition of Ratio In certain situations, the comparison of two quantities by the method of division is very efficient. We can say that the comparison or simplified form of two quantities of the same kind is referred to as ratio. This relation gives us how many times one quantity is equal to the other quantity. In simple words, the ratio is the number which can be used to express one quantity as a fraction of the other ones.



The two numbers in a ratio can only be compared when they have the same unit. We make use of ratios to compare two things.

The sign used to denote a ratio is ':'.
A ratio can be written as a fraction, say $\frac{2}{5}$. We happen to see various comparisons or say ratios in our daily life.

Key Points to Remember:

The ratio should exist between the quantities of the same kind While comparing two things, the units should be similar There should be significant order of terms

The comparison of two ratios can be performed, if the ratios are equivalent like the fractions

Definition of Proportion

Proportion is an equation which defines that the two given ratios are equivalent to each other. In other words, the proportion states the equality of the two fractions or the ratios. In proportion, if two sets of given numbers are increasing or decreasing in the same ratio, then the ratios are said to be directly proportional to each other.

For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of

500km for 5 hours. Such as $100\text{km/hr} = 500\text{km}/5\text{hrs}$.

Ratio and proportions are said to be faces of the same coin. When two ratios are equal in value, then they are said to be in proportion. In simple words, it compares two ratios. Proportions are denoted by the symbol '::' or '='.

Continued Proportion

Consider two ratios to be a: b and c: d.

Then in order to find the continued proportion for the two given ratio terms, we convert the means to a single term/number.

This would, in general, be the LCM of means.

For the given ratio, the LCM of b & c will be bc.

Thus, multiplying the first ratio by c and second ratio by b, we have

First ratio- ca:bc

Second ratio- bc: bd

Thus, the continued proportion can be written in the form of ca: bc: bd

Also, read:



Direct And Inverse Proportion

Ratio To Percentage

Ratio and Proportion Formula

Now, let us learn the Maths ratio and proportion formulas here.

Ratio Formula

Assume that, we have two quantities (or two numbers or two entities) and we have to find the ratio of these two,

then the formula for ratio is defined as;

$$a : b \Rightarrow a/b$$

where a and b could be any two quantities.

Here, “a” is called the first term or antecedent, and “b” is called the second term or consequent.

Example: In ratio 4:9, is represented by $4/9$, where 4 is antecedent and 9 is consequent.

If we multiply and divide each term of ratio by the same number (non-zero), it doesn't affect the ratio.

Example: $4:9 = 8:18 = 12:27$

Also, read: Ratio Formula

Proportion Formula

Now, let us assume that, in proportion, the two ratios are a:b & c:d. The two terms 'b' and 'c' are called 'means or

mean term,' whereas the terms 'a' and 'd' are known as 'extremes or extreme terms.'

$$a/b = c/d \text{ or } a : b :: c : d$$

Example: Let us consider one more example of a number of students in a classroom. Our first ratio of the number of

girls to boys is 3:5 and that of the other is 4:8, then the proportion can be written as:

$$3 : 5 :: 4 : 8 \text{ or } 3/5 = 4/8$$

Here, 3 & 8 are the extremes, while 5 & 4 are the means.



Note: The ratio value does not affect when the same non-zero number is multiplied or divided on each term.

Important Properties of Proportion

The following are the important properties of proportion:

Addendo – If $a : b = c : d$, then $a + c : b + d$

Subtrahendo – If $a : b = c : d$, then $a - c : b - d$

Dividendo – If $a : b = c : d$, then $a - b : b = c - d : d$

Componendo – If $a : b = c : d$, then $a + b : b = c + d : d$

Alternendo – If $a : b = c : d$, then $a : c = b : d$

Invertendo – If $a : b = c : d$, then $b : a = d : c$

Componendo and dividendo – If $a : b = c : d$, then $a + b : a - b = c + d : c - d$

Difference Between Ratio and Proportion

To understand the concept of ratio and proportion, go through the difference between ratio and proportion given

here.

S.No Ratio Proportion

1 The ratio is used to compare the size of two things with the same unit The proportion is used to express the

relation of two ratios

2 It is expressed using a colon (:), slash (/) It is expressed using the double colon (::) or equal to the symbol (=)

3 It is an expression It is an equation

4 Keyword to identify ratio in a problem is “to every” Keyword to identify proportion in a problem is “out

of”



Fourth, Third and Mean Proportional

If $a : b = c : d$, then:

d is called the fourth proportional to a, b, c .

c is called the third proportion to a and b .

Mean proportional between a and b is \sqrt{ab} .

Comparison of Ratios

If $(a:b) > (c:d) = (a/b > c/d)$

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

Duplicate Ratios

If $a:b$ is a ratio, then:

$a^2:b^2$ is a duplicate ratio

$\sqrt{a}:\sqrt{b}$ is the sub-duplicate ratio

$a^3:b^3$ is a triplicate ratio

Ratio and Proportion Tricks

Let us learn here some rules and tricks to solve problems based on ratio and proportion topic.

If $u/v = x/y$, then $uy = vx$

If $u/v = x/y$, then $u/x = v/y$

If $u/v = x/y$, then $v/u = y/x$

If $u/v = x/y$, then $(u+v)/v = (x+y)/y$

If $u/v = x/y$, then $(u-v)/v = (x-y)/y$

If $u/v = x/y$, then $(u+v)/(u-v) = (x+y)/(x-y)$, which is known as componendo -Dividendo Rule

If $a/(b+c) = b/(c+a) = c/(a+b)$ and $a+b+c \neq 0$, then $a = b = c$



Unit:2

Profit and Loss:

Profit and Loss :

Profit and Loss formula is used in mathematics to determine the price of a commodity in the market and understand how profitable a business is. Every product has a cost price and selling price. Based on the values of these prices, we can calculate the profit gained or the loss incurred for a particular product. The important terms covered here are cost price, fixed, variable and semi-variable cost, selling price, marked price, list price, margin, etc.

Also, we will learn the profit and loss percentage formula here. For example, for a shopkeeper, if the value of selling price is more than the cost price of a commodity, then it is a profit and if the cost price is more than the selling price, it becomes a loss. Here, in this article, we will discuss profit as well as loss concepts along with tricks to solve problems based on it.

Profit and Loss Basic Concepts Let us learn profit and loss concepts in maths.

It is well explained in terms of cost price and selling price.

Profit(P) The amount gained by selling a product with more than its cost price.

Loss(L) The amount the seller incurs after selling the product less than its cost price, is mentioned as a loss.

Cost Price (CP) The amount paid for a product or commodity to purchase it is called a cost price. Also, denoted as CP. This cost price is further classified into two different categories:

Fixed Cost: The fixed cost is constant, it doesn't vary under any circumstances

Variable Cost: It could vary depending as per the number of units Selling Price (SP)

The amount for which the product is sold is called Selling Price. It is usually denoted as SP. Also, sometimes called a sale price.

Marked Price Formula (MP) This is basically labelled by shopkeepers to offer a discount to the customers in such a way that,

Discount = Marked Price – Selling Price And Discount Percentage = $(\text{Discount}/\text{Marked price}) \times 100$ Profit and Loss



Formulas Now let us find profit formula and loss formula. The profit or gain is equal to the selling price minus cost price. Loss is equal to cost price minus selling price.

$$\text{Profit or Gain} = \text{Selling price} - \text{Cost Price}$$

$$\text{Loss} = \text{Cost Price} - \text{Selling Price}$$

The formula for the profit and loss percentage is:

$$\text{Profit percentage} = (\text{Profit} / \text{Cost Price}) \times 100$$

$$\text{Loss percentage} = (\text{Loss} / \text{Cost price}) \times 100$$

Examples

If a shopkeeper brings a cloth for Rs.100 and sells it for Rs.120, then he has made a profit of Rs.20/-. If a salesperson has bought a textile material for Rs.300 and he has to sell it for Rs.250/-, then he has gone through a loss of Rs.50/-.

Suppose, Ram brings a football for Rs. 500/- and he sells it to his friend for Rs. 600/-, then Ram has made a profit of Rs.100 with the gain percentage of 20%.

These are some common examples of the profit and loss concept in real life, which we observe regularly.

Profit and Loss Tricks

You have learned until now how to calculate profit as well as loss and also the percentage of them. Now let us learn some tricks or formulas to solve maths problems based on gain and loss, starting from the general formulas

$$\text{Profit, } P = SP - CP; SP > CP$$

$$\text{Loss, } L = CP - SP; CP > SP$$

$$P\% = (P/CP) \times 100$$

$$L\% = (L/CP) \times 100$$

$$SP = \{(100 + P\%)/100\} \times CP$$

$$SP = \{(100 - L\%)/100\} \times CP$$

$$CP = \{100/(100 + P\%)\} \times SP$$



$$CP = \{100/(100 - L\%)\} \times SP$$

$$\text{Discount} = MP - SP$$

$$SP = MP - \text{Discount}$$

For false weight, profit percentage will be $P\% = (\text{True weight} - \text{false weight} / \text{false weight}) \times 100$.

When there are two successful profits say $m\%$ and $n\%$, then the net percentage profit equals to $(m+n+mn)/100$

When the profit is $m\%$ and loss is $n\%$, then the net % profit or loss will be: $(m-n-mn)/100$

If a product is sold at $m\%$ profit and then again sold at $n\%$ profit then the actual cost price of the product will be: $CP =$

$$[100 \times 100 \times P / (100+m)(100+n)]. \text{ In case of loss, } CP = [100 \times 100 \times P / (100-m)(100-n)]$$

If $P\%$ and $L\%$ are equal then, $P = L$ and $\% \text{loss} = P^2/100$

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Unit:3

Interest and Annuity:

Q:1) What day of the week does May 28 2006 fall on

Brokerage:

We have studied so far selling and buying of products. Let us study the providing of services. One such service provider is Real Estate Agency: The agency gets requests from those who want to buy lands, sites, houses, shops. Or those who require shops/houses on rent..

He is also approached by people who want to sell sites, houses, shops and others who are looking for tenants.

The real estate agency acts as 'middle man' between 'buyers' and 'sellers'.

They charge 'brokerage' or 'commission' for providing the services.

Same is the case with agencies who deal with buying and selling of used cars.

You also must have heard about marriage bureaus. They provide services in finding matches for brides and bridegrooms.

Another service which is popular in major cities is 'Placement services'

They collect details about people who seek employments in companies. These agencies help the candidates in getting suitable employment in companies. For their service they normally charge 10% of employee's salary. This is paid by companies and not by job seekers.

The amount (fee) received by the agency for its services are called 'commission' or 'brokerage'.

It is not that this type of business is carried out only by an agency always. An individual can also carry out this type of business

An individual who acts as a 'middleman' between a buyer and seller is called a 'broker' or 'commission agent

The commission or brokerage need not always be a fixed % of sale price; it can also be a fixed amount as in the example:

'One month rent is paid as brokerage for identifying premises for a person who is in need of a house/shop on rent' Generally, Brokerage is paid by purchaser, Commission is paid by seller.



Therefore Net amount paid by purchaser = Sale price+brokerage

Net amount received by seller = Sale price - commission

Like in case of discount,

we have similar formula:

Commission (Brokerage) = $\text{Commission\%} \times \text{Selling price} / 100$

From this formula we can arrive at formulae for

Commission % = $\text{Commission} \times 100 / \text{Selling price}$

Selling Price = $\text{Commission} \times 100 / \text{Commission \%}$

Simple and Compound Interest :

Interest Formula Interest formulas mainly refer to the formulas of simple and compound interests. The simple interest (SI) is a type of interest that is applied to the amount borrowed or invested for the entire duration of the loan, without taking any other factors into account, such as past interest (paid or charged) or any other financial considerations. Simple interest is generally applied to short-term loans, usually one year or less, that are administered by financial companies. The same applies to money invested for a similarly short period of time. The simple interest rate is a ratio and is typically expressed as a percentage.

On the other hand, the compound interest is the interest which is calculated on the principal and the interest that is accumulated over the previous tenure. Thus, the compound interest (CI) is also called as “interest on interest”. It plays an important role in determining the amount of interest on a loan or investment. The formulas for both the compound and simple interest is given below.

Interest Formulas for SI and CI

The Interest formulas are given as,

Formulas for Interests (Simple and Compound)

SI Formula S.I. = $\text{Principal} \times \text{Rate} \times \text{Time}$

CI Formula C.I. = $\text{Principal} (1 + \text{Rate})^{\text{Time}} - \text{Principal}$

Example Problem Using Interest Formulas



Unit:4

Matrices and Determinant:

Determinants and Matrices

Determinants and matrices, in linear algebra, are used to solve linear equations by applying Cramer's rule to a set of non-homogeneous equations which are in linear form. Determinants are calculated for square matrices only. If the determinant of a matrix is zero, it is called a singular determinant and if it is one, then it is known as unimodular. For the system of equations to have a unique solution, the determinant of the matrix must be nonsingular, that is its value must be nonzero. In this article, let us discuss the definition of determinants and matrices, different matrices types, properties, with examples.

Matrices Definition

Matrices are the ordered rectangular array of numbers, which are used to express linear equations. A matrix has rows and columns. we can also perform the mathematical operations on matrices such as addition, subtraction, multiplication of matrix. Suppose the number of rows is m and columns is n , then the matrix is represented as $m \times n$ matrix.

Also, read:

Inverse Of 3 By 3 Matrix

Identity Matrix

Types of Matrices

There are different types of matrices. Let's see some of the examples of different types of matrices

Inverse of a Matrix

Inverse of a matrix is defined usually for square matrices. For every $m \times n$ square matrix, there exists an inverse matrix. If A is the square matrix then A^{-1} is the inverse of matrix A and satisfies the property

$AA^{-1} = A^{-1}A = I$, where I is the Identity matrix.

Also, the determinant of the square matrix here should not be equal to zero.

Transpose of Matrix

The transpose of a matrix can be determined by rows for the columns. If A is a matrix, then the transpose of a matrix is represented by A^T .



Definition of Determinant

A determinant can be defined in many ways for a square matrix.

The first and most simple way is to formulate the determinant by taking into account the top row elements and the corresponding minors. Take the first element of the top row and multiply it by its minor, then subtract the product of the second element and its minor.

Continue to alternately add and subtract the product of each element of the top row with its respective minor until all the elements of the top row have been considered.

For example let us consider a 4×4 matrix A.

determinant of matrix

Second Method to find the determinant:

The second way to define a determinant is to express in terms of the columns of the matrix by expressing an $n \times n$ matrix in terms of the column vectors.

Consider the column vectors of matrix A as $A = [a_1, a_2, a_3, \dots, a_n]$

where any element a_j is a vector of size x .

Then the determinant of matrix A is defined such that $\text{Det} [a_1 + a_2 \dots b_j + c v \dots a_x] = b$

$\text{det}(A) + c \text{det} [a_1 + a_2 + \dots v \dots a_x]$

$\text{Det} [a_1 + a_2 \dots a_j a_{j+1} \dots a_x] = -\text{det} [a_1 + a_2 + \dots a_{j+1} a_j \dots a_x]$

$\text{Det}(I) = 1$ Where the scalars are denoted by b and c , a vector of size x is denoted by v , and the identity matrix of size x is denoted by I . We can infer from these equations that the determinant is a linear function of the columns. Further, we observe that the sign of the determinant can be interchanged by interchanging the position of adjacent columns. The identity matrix of the respective unit scalar is mapped by the alternating multi-linear function of the columns. This function is the determinant of the matrix.

Check: Determinant Of A 3×3 Matrix

Properties of Determinant

If I_n is the identity matrix of the order $n \times n$, then $\text{det}(I) = 1$ If the matrix M^T is the transpose of matrix M , then $\text{det}(M^T) = \text{det}(M)$ If matrix M^{-1} is the inverse of matrix M , then $\text{det}(M^{-1}) = \frac{1}{\text{det}(M)} = \text{det}(M)^{-1}$

If two square matrices M and N have the same size, then $\text{det}(MN) = \text{det}(M) \text{det}(N)$ If matrix M has a size $a \times a$ and C is a constant, then $\text{det}(CM) = C^a \text{det}(M)$

If X , Y , and Z are three positive semidefinite matrices of equal size, then the following holds true along with the corollary $\text{det}(X+Y) \geq \text{det}(X) + \text{det}(Y)$ for $X, Y, Z \geq 0$ $\text{det}(X+Y+Z) + \text{det} C \geq \text{det}(X+Y) + \text{det}(Y+Z)$ In a triangular matrix, the determinant is equal to the product of the diagonal elements.

The determinant of a matrix is zero if all the elements of the matrix are zero. Laplace's Formula and the Adjugate Matrix Apart from these properties of determinants, there are some other properties.



Unit:5

Linear Programming Problem (LPP)

Linear Programming Problems (LPP):

Linear programming or linear optimization is a process which takes into consideration certain linear relationships to obtain the best possible solution to a mathematical model. It is also denoted as LPP. It includes problems dealing with maximizing profits, minimizing costs, minimal usage of resources, etc. These problems can be solved through the simplex method or graphical method.

The Linear programming applications are present in broad disciplines such as commerce, industry, etc. In this section, we will discuss, how to do the mathematical formulation of the LPP.

Mathematical Formulation of Problem Let x and y be the number of cabinets of types 1 and 2 respectively that he must manufacture.

They are non-negative and known as non-negative constraints. The company can invest a total of 540 hours of the labour force and is required to create up to 50 cabinets.

$$\text{Hence, } 15x + 9y \leq 540$$

$$x + y \leq 50$$

The above two equations are known as linear constraints.

Let Z be the profit he earns from manufacturing x and y pieces of the cabinets of types 1 and 2.

$$\text{Thus } Z = 5000x + 3000y$$

Our objective here is to maximize Z . Hence Z is known as the objective function. To find the answer to this question, we use graphs, which is known as the graphical method of solving LPP. We will cover this in the subsequent sections.

Graphical Method

The solution for problems based on linear programming is determined with the help of the feasible region, in case of graphical method. The feasible region is basically the common region determined by all constraints including non negative constraints, say, $x, y \geq 0$, of an LPP. Each point in this feasible region represents the feasible solution of the constraints and therefore, is called the solution/feasible region for the problem. The region apart from (outside) the feasible region is called as the infeasible region.



The optimal value (maximum and minimum) obtained of an objective function in the feasible region at any point is called an optimal solution. To learn the graphical method to solve linear programming completely reach us.

Linear Programming Applications

Let us take a real-life problem to understand linear programming. A home décor company received an order to manufacture cabinets. The first consignment requires up to 50 cabinets. There are two types of cabinets. The first type requires 15 hours of the labour force (per piece) to be constructed and gives a profit of Rs 5000 per piece to the company. Whereas, the second type requires 9 hours of the labour force and makes a profit of Rs 3000 per piece. However, the company has only 540 hours of workforce available for the manufacture of the cabinets. With this information given, you are required to find a deal which gives the maximum profit to the décor company.

Given the situation, let us take up different scenarios to analyse how the profit can be maximized.

He decides to construct all the cabinets of the first type. In this case, he can create $540/15 = 36$ cabinets. This would

give him a profit of $\text{Rs } 5000 \times 36 = \text{Rs } 180,000$.

He decides to construct all the cabinets of the second type. In this case, he can create $540/9 = 60$ cabinets. But the

first consignment requires only up to 50 cabinets. Hence, he can make profit of $\text{Rs } 3000 \times 50 = \text{Rs } 150,000$

He decides to make 15 cabinets of type 1 and 35 of type 2. In this case, his profit is $(5000 \times 15 + 3000 \times 35)$ Rs 180,000.

Similarly, there can be many strategies which he can devise to maximize his profit by allocating the different amount of labour force to the two types of cabinets. We do a mathematical formulation of the discussed LPP to find out the strategy which would lead to maximum profit.

Different Type of Linear Programming Problems

The various types of problem in linear programming problem included in class 12 concepts.

They are:

- (i) Manufacturing problem- Here we maximize the profit with the help of minimum utilization of the resource.



- (ii) Diet Problem- We determine the number of different nutrients in a diet to minimize the cost of manufacturing
- (iii) Transportation problem- Here we determine the schedule to find the cheapest way of transporting a product at minimum time.

Terms of Linear Programming Problem There exist specific terms (or terminology) while constructing and solving linear programming problems. Let us define some of the important terms which we shall be using here

Objective function: A linear function of the form $Z = ax + by$, where a and b are constant, which has to be minimized or maximized is called a linear objective function. Consider an example, $Z = 175x + 150y$. This is a linear objective function. The variables x and y are called decision variables

Constraints: The linear inequalities or equations or restrictions on the variables of LPP (linear programming problem) are called constraints. The conditions $x \geq 0$, $y \geq 0$ are called non-negative restrictions. For example, $5x + y \leq 100$; $x + y \leq 60$ are constraints.

Optimization problem: A problem which seeks to maximize or minimize a linear function (say of two variables x and y) subject to certain constraints as determined by a set of linear inequalities is called an optimization problem. Linear programming problems are special types of optimization problems.

Feasible region: The common region determined by all the given constraints including non-negative constraints ($x \geq 0$, $y \geq 0$) of a linear programming problem is called the feasible region (or solution region) for the problem. The region other than feasible is called an infeasible region.

Feasible solutions: These are the points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is called an infeasible solution.

Optimal (or feasible) solution: Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution

Note: A corner point of a feasible region is the point of intersection of two boundary lines, which form the region. A feasible region of a given system of linear inequalities is said to be bounded if it can be enclosed within a circle. Otherwise, it is unbounded. Unbounded means that the feasible region may extend indefinitely in any direction. Linear Programming Class 12 Sample Problem Question: Solve the following linear programming problem graphically

Minimize $Z = 200x + 500y$

subject to the constraints: $x + 2y \geq 10$



$$3x + 4y \leq 24$$

$$x \geq 0, y \geq 0$$

Solution:

Given objective function is:

$$\text{Minimize } Z = 200x + 500y \dots(i)$$

Constraints are:

$$x + 2y \geq 10 \dots(ii)$$

$$3x + 4y \leq 24 \dots(iii)$$

$$x \geq 0, y \geq 0 \dots(iv)$$

The shaded region in the figure (above graph) is the feasible region ABC determined by the system of constraints (ii),

(iii) and (iv), which is bounded.

The coordinates of corner points of this (feasible or shaded) region say A, B and C are (0, 5), (4, 3) and (0, 6)

respectively.

Now, let us evaluate the value of $Z = 200x + 500y$ at these points.

Corner point Corresponding value of Z

$$(0, 5) \quad 200 \times 0 + 500 \times 5 = 0 + 2500 = 2500$$

$$(4, 3) \quad 200 \times 4 + 500 \times 3 = 800 + 1500 = 2300 \text{ (minimum)}$$

$$(0, 6) \quad 200 \times 0 + 500 \times 6 = 0 + 3000 = 3000$$

Hence, the minimum value of Z is 2300 at the point (4, 3)