Unit 1 Basic Structure and Introduction to Data structure

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Introduction to Data structure

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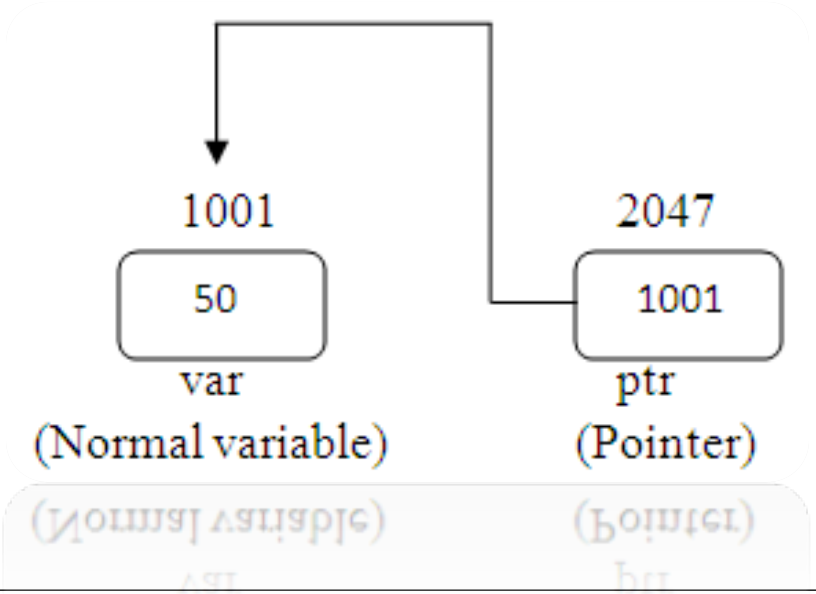
1.1 Pointers and Dynamic Memory allocation

Pointer:

A pointer is used as a referencing mechanism a pointer provides a way to reference an object using that object's address. There are usually three elements involved in this referencing process, a pointer variable, an address, and another variable .The pointer variable holds the address of the other variable. A special operation, called an indirection operation will use this address to actually reference the other variable,

a) A normal variable 'var' has a memory address of 1001 and holds a value 50.

b) A pointer variable has its own address 2047 but stores 1001, which is the address of the variable 'var'.



Working with pointers:

1) Declaration of a Pointer:

The declaration of a pointer variable takes the following form:

data type *pt_name; This tells the compiler three things about the variable pt_name:

- The asterisk (*) tells that the variable pt_name is a pointer variable.
- pt_name needs a memory location.
- pt_name points to a variable of type data type.

2) Initialization of Pointer variables:

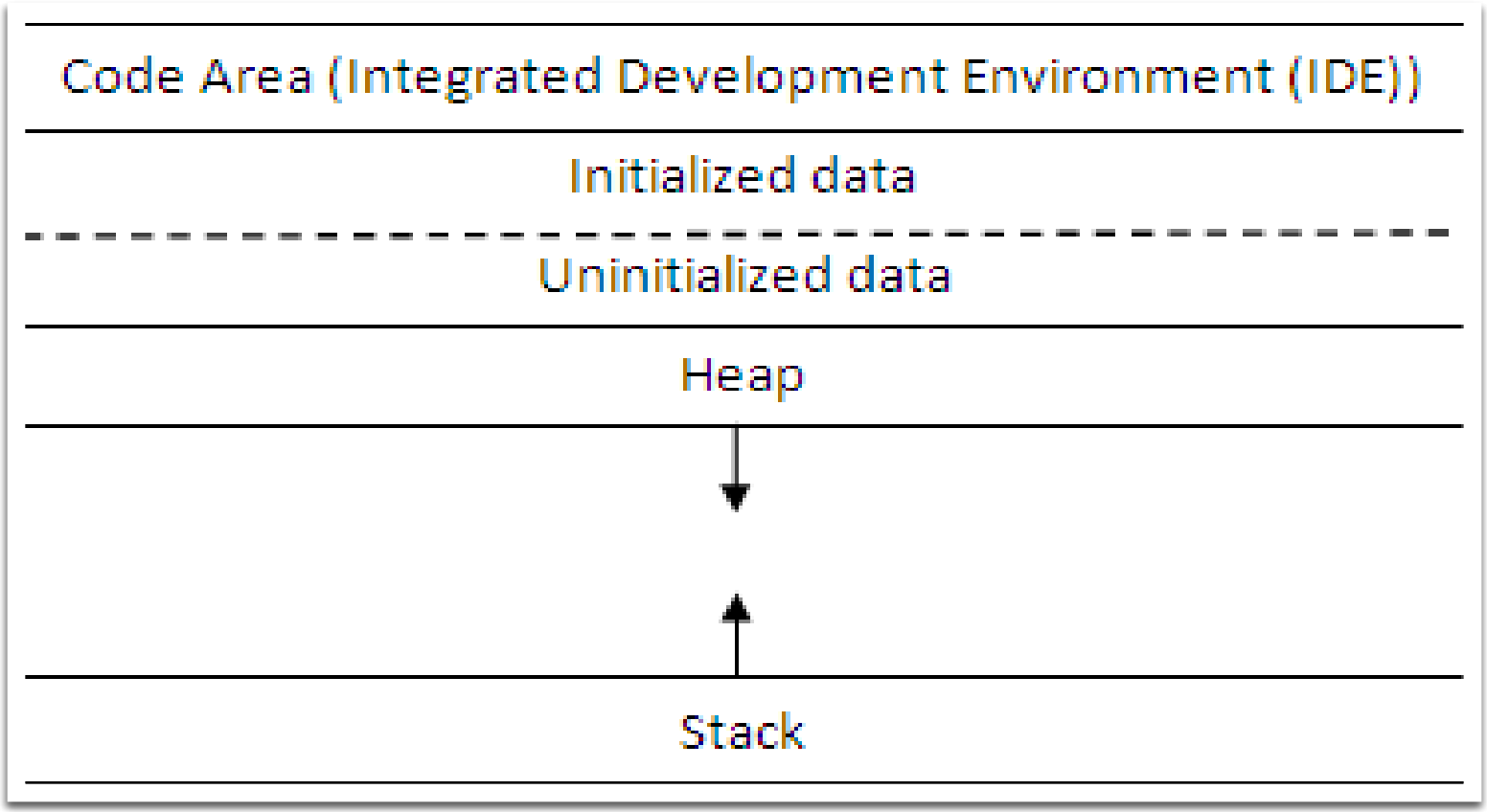
The initialization of the pointer variable is simple like other variable but in the pointer variable the address is assigned to the pointer variable instead of value.

3) Accessing a Variable through its Pointer:

Once a pointer has been assigned the address of a variable, one can access the value of the variable using unary operator '*'(asterisk), known as the indirection operator or dereferencing operator.

C) Dynamic Memory Allocation:

The technique through which a program can be obtaining space in the RAM during the execution of the program and not during compilation is called dynamic memory allocation. The entire runtime view of memory for a program is given below:



Functions of Dynamic Memory Allocation:

1) malloc() function:

malloc() function is used for allocating block of memory at runtime. This function reserves a block of memory of given size and returns a pointer of typo void. This means that one can assign it to any type of pointer using typecasting. If it fails to locate enough space it returns a NULL pointer.

2) calloc() function:

calloc() is another memory allocation function that is used for allocating memory at runtime. calloc() initializes the allocated memory to zero but, malloc() doesn't. calloc() function is normally used for allocating memory to derived data types such as arrays and structures. If it fails to locate enough space it returns a NULL pointer.

3) realloc() function:

realloc () function modifies the allocated memory size by malloc () and calloc () functions to new size. If enough space doesn't exist in memory of current block to extend, new block is allocated for the full size of reallocation, then copies the existing data to new block and then frees the old block.

4) free() function:

When a program comes out, operating system automatically release all the memory allocated by the program but as a good practice when there is no need of memory anymore then the memory should be released by calling the function free().free() function frees the allocated memory by malloc (), calloc (), realloc () functions and returns the memory to the system.

1.2 Algorithm-Definition and characteristics

Definitions:

1) Alonzo Church and Alan Turing:

"An algorithm is defined as the finite sequence of instructions, each of which has a clear meaning and can be performed with a finite amount of effort in a finite length of time."

2) David Hilbert:

"An algorithm is a set of instructions designed to perform a specific task."

Characteristics of Algorithm:

1) Finiteness:

An algorithm must terminate after a finite number of steps.

2) Definiteness:

The steps of the algorithm must be precisely defined or unambiguously specified.

3) Correctness:

The output must be true for all input values.

4) Generality:

An algorithm must be generic enough to solve all problems of a particular class.

1.3 Algorithm Analysis :
Space Complexity & Time Complexity

Analysis of Algorithm:

1) Considerations in Algorithm Analysis:

Analysis of algorithms focuses on computation of space and time complexity. Space can be defined in terms of space required to store the instructions and data whereas the time is the computer time an algorithm might require for its execution which usually depends on the size of the algorithm and input.

a) Space Complexity:

The space complexity of a problem is a related concept that measures the amount of space, or memory required by the algorithm. Space complexity is measured with Big-O notation.

b) Time Complexity:

Time Complexity is defined as the computer time an algorithm might require for its execution, which usually depends on the size of the algorithm and input.

2) Types of Time Complexities:

a) Best Case Time Complexity:

The best case time complexity of an algorithm is a measure of the minimum time that the algorithm will require for an input of size ‘n`.

b) Worst Case Time Complexity:

The worst case time complexity of an algorithm is a measure of the maximum time that the algorithm will require for an input of size ‘n'.

c) Average Case Time Complexity:

The time that an algorithm will require to execute a typical input data of size ‘n' is known as average case time complexity

Asymptotic Notations:

Asymptotic notation is an easiest way to write down or describe the running time of an algorithm.

These asymptotic notations are as follows:

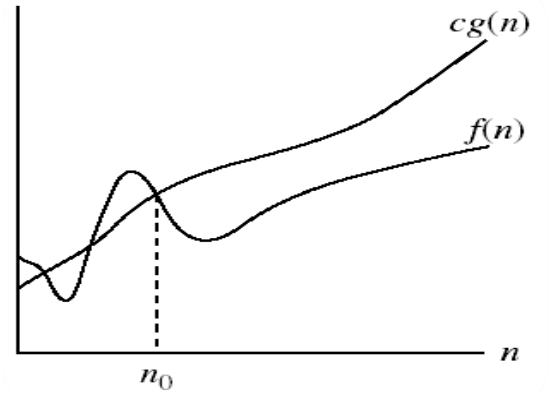
i) Big-oh or "O"-Notation:

O-notation is used to expressing the upper bound of an algorithms running time.

g(n) is a given function, O (g (n)) is a set of function n, then it is given as:

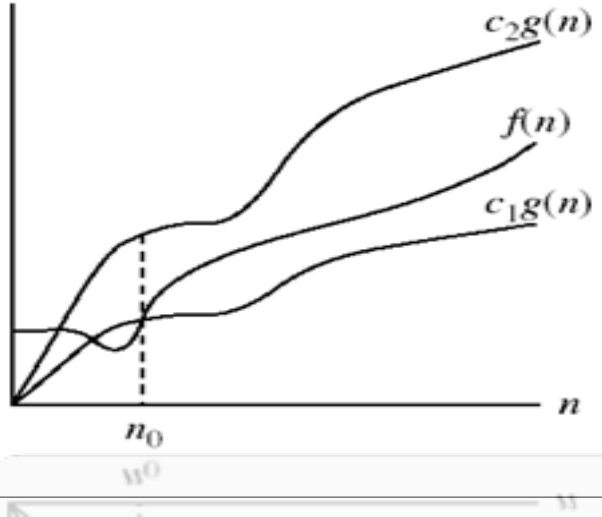
O(g(n)) = { f(n) : there exist positive constants c and n_0 such that ,

$0 \leq f(n) \leq c*g(n)$, for all $n \geq n_0$ } .



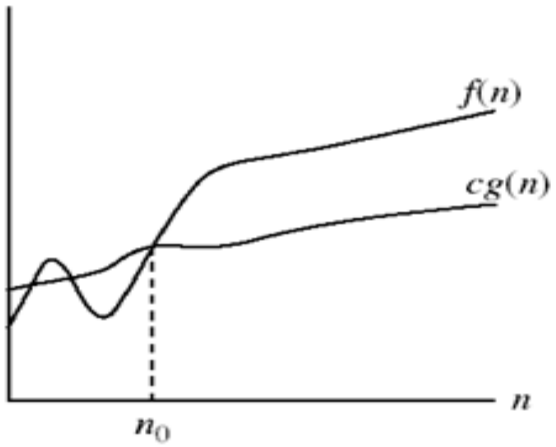
ii) θ-notation:

g (n) is a given function and θ(g (n)) is the set of functions then, (g (n)) = { f(n) : there exist positive constants c_1, c_2 and n_0 such that , $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$,for all $n \geq n_0$ } .



iii) Ω-notation:

Ω-notation is used for asymptotic lower bound. For a given function g (n),Ω(g (n)) as the set of functions ,Ω (g(n)) = { f (n) : there exist positive constants c and n_0 such that $0 \leq c * g(n) \leq f(n)$, for all $n \geq n_0$ }



iv) o-notation or 'little oh':

o-notation is used to denote an upper bound that is not asymptotically tight. It is also called 'little oh'.o(g (n)) = { f(n) : For any positive constant c >0, there exists constant n_0> 0 such that $0 \leq f(n) \leq cg(n)$, for all $n \geq n_0$ }.The function f (n) becomes insignificant relative to g(n) as n approaches infinity,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

v) Little-Omega Notation:

It is denoted by ω-notation. ω- Notation is used to denote a lower bound that is not asymptotically tight (g (n)) = {f (n): For any positive constant c > 0, if a constant n_0> 0 such that, $0 \leq cg(n) \leq f(n)$, for all $n \geq n_0$ }.

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

Introduction to Data Structure

- A data structure in Computer Science, is a way of storing and organizing data in a computer's memory or even disk storage so that it can be used efficiently.
- A well-designed data structure allows a variety of critical operations to be performed.
- Data structures are implemented by programming language by the data types, references and operations provide by that particular language.
- Different kinds of data structures are suited to different kinds of applications, and some are highly specialized to certain tasks.

1.5 DATASTRUCTURE AND ITS TYPES

Basically, data structures are of two types

- linear data structure and non linear data structure.

1. Linear data structure :

A data structure is said to be linear if the elements form a sequence i.e., while traversing sequentially, we can reach only one element directly from another.

For example : Array, Linked list,Queue etc.

2. Non linear data structure :

Elements in a nonlinear data structure do not form a sequence i.e each item or element may be connected with two or more other items or elements in a non-linear arrangement. **For example :** Trees and Graphs etc.

DATA STRUCTURE OPERATIONS

- We come to know that data structure is used for the storage of data in computer so that data can be used efficiently.
- The data manipulation within the data structures are performed by means of certain operations.

The following four operations play a major role on data structures.

a) Traversing : Accessing each record exactly once so that certain items in the record may be processed. (This accessing and processing is sometimes called “visiting” the record.)

b) Searching : Finding the location of the record with a given key value, or finding the locations of all records, which satisfy one or more conditions.

c) Inserting : Adding a new record to the structure.

d) Deleting : removing a record from the structure.

Sometimes two or more of these operations may be used in a given situation.

For example, if we want to delete a record with a given key value, at first we will have need to search for the location of the record and then delete that record.

The following two operations are also used in some special situations :

i) Sorting : Operation of arranging data in some given order, such as increasing or decreasing, with numerical data, or alphabetically, with character data.

ii) Merging : combining the records in two different sorted files into a single sorted file.

1.5 Abstract Data Type

A) Meaning:

- An Abstract Data Type (ADT) is a mathematical model of the data objects that make up a data type as well as the functions that operate on these objects.
- An abstract data type is the specification of logical and mathematical properties of a data type or structure.
- ADT acts as a useful guideline to implement a data type correctly. The specification of an ADT does not imply any implementation consideration.
- The implementation of an ADT involves the translation of the ADT`s specification into syntax of a particular programming language.
- Thus, ADT involves mainly two parts:
  a) Description of the way in which components are related to each other.
  b) Statements of operations that can be performed on that data

Array : Introduction

- The fundamental data types namely int, float, char etc. are very useful but variable of these data type can be stored only one value at a time. So they can handle limited amount of data. In many applications we need to handle large volume of data for that we have to need powerful data types that would facilitate efficient storing, accessing and manipulation of data items.
- C supports derived data type known as Array.
- Definition-
- An array is collection of data items of the same data type
- An array is fixed-size sequenced collection of elements of the same data type.
- An array is also called as Subscripted Variables
-

Features of Array:

- An array is a collection of similar elements.
- The location of array is the location of its first element.
- The first element in array is numbered zero so the last element is less than the size of array.
- The length of array is the number of its elements in array.
- The type of an array is the data type of its element.
- An array is known as subscripted variable.
- Before using array it's type and dimension must be declared.

1.6 Types of Array 1.Single Dimension Array :

A list of items can be given one variable name using only one subscript and such a variable is called a single subscripted or single Dimension Array.

Syntax :

data type arrayname[size];

Example :

int rollno[3];
float marks[5];
char name[30];

1340 1342 1344 ----- Memory Address

Array elements are always stored in contiguous memory locations and since the data type int occupies 2 bytes of memory, each element will be allocated 2 bytes.

Declaration and Initialization Array :

We can initialise elements of array in the same way the ordinary variable.

Syntax :

data type arrayname[size]={list of values};

Example :

int rollno[5]={1,2,3,4,5}; int rollno[ ]={1,2,3,4,5};
float num[5]={2.5,7.2,9.2,6.2,3.3};

2. Multi Dimension Array :

An array whose elements are speacified by more than one subscript is known as multi dimension array (also called Matrix)

Syntax :

data type arrayname[row size][column size];

Example :

int student[5][2];

	C0	C1
R0	1	67
R1	2	73
R2	3	82
R3	4	90
R4	5	58

char name [4][10];

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9
R0	S	W	A	P	N	I	L	\0		
R1	S	A	N	T	O	S	H	\0		
R2	J	A	Y	D	I	P	\0			
R3	S	A	N	D	I	P	\0			

Declaration and Initialisation Array :

int number[3][4]={8,12,25,37,42,52,68,79,81,92,100,102};

char city[5][10]={“Mumbai”,”Punei”,”Satara”,”Kolhapur”,”Sangli”};

String (Character Array) :

In C character string simply treats as array character. The size in a character string represents the maximum number of characters that the string can hold.

Example :

char name[10];

It declares the name as a character array (string) variable that can hold a maximum 10 characters. Each character of the string is treated as an element of the array name and is stored in the memory as follows.

A Character string terminates with an additional null character. Thus the element in name[10] holds the null character '\0'. When declaring character arrays, we must allow one extra element space for the null terminator.

Declaration and Initialisation Array

We can initialise elements of array in the same way as the ordinary variable.

Syntax :

data type arrayname[size]={list of values};

Example :

```
char name[5]={'s','w','a','p','n','i','l','\0'};
char name[]="siddhi";
char name[5]={'P'};
char  *colour[]={"Red","Green","Blue","Yellow"};
```

1.7 Polynomial Representation:

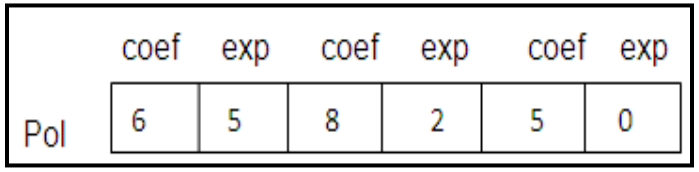
A polynomial of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0$ can be considered as a list comprising coefficients and exponents as shown below:

$F(x) = \{ a_n x^n , a_{n-1} x^{n-1} , \ldots . a_1 x^1 , a_{0,} x^0 \}$

For example, the polynomial $6x^5 + 8x^2 + 5$ can be represented as the list shown below:

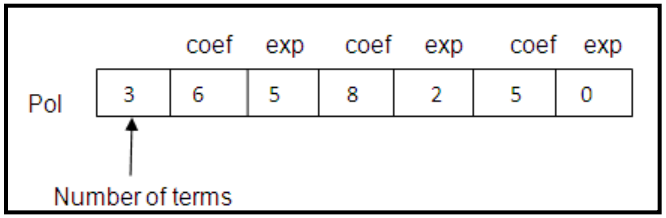
Polynomial = {6, 5, 8, 2, 5, 0}

The above list can be very easily implemented using a one-dimensional array, say Pol [ ] as shown in below figure., where every alternate location contains coefficient and exponent.



Polynomials Representation using Array

The above representation can be modified to incorporate number of terms present in a polynomial by reserving the 0^{th}location of the array for this purpose. The modified representation is shown in below figure. A general polynomial can be represented in an array with descending order of its degree of terms. Therefore, the operations such as addition, multiplication and division on polynomials can be easily carried out.



0^{th}place Reserved for Number of Terms

- **Evolution of Polynomial:**

A generic polynomial is of the form: $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots a_2 x^2 + a_1 x + a_0$. It is assumed that the coefficient values of a_0 through a_n are all known and constant and will be stored in an array. Thus, the evaluation of a polynomial has only the value of x as its input and will return the resulting polynomial value as its output. An alternative method of writing the polynomial is :

$$p(x)\ x = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{n-1} x^{n-1} + a_n x^n$$

$$= a_0 + x\,(a_1 + x\,(a_2 + x\,(a_3 + x\,(a_4 + \ldots . x(a_{n-1} + x\,a_n)))))$$

The method requires only n multiplication and n additions. The polynomial evaluation can be performed by evaluating the expression in the innermost parenthesis and successfully multiplying by x in for loop. The coefficients $a_0, a_1, a_2, \ldots a_n$ are stored in an array a[n].

Addition of Polynomial:

When adding polynomials only the coefficients of same powerare added and subtracted, the exponents remain unchanged. While adding two polynomials, following cases need to be considered.

1) When the Degrees of Corresponding Terms of the Two Polynomials are Same:

This is the normal case when corresponding coefficients of each term can be added directly.

Example:

$5x^3+2x^2+7$

$7x^3+9x^2+12$

$12x^3+11x^2+19$ is a simple addition where all the degrees of the corresponding terms are same.

2) When the degrees of corresponding terms of the polynomials are different:

The terms with of same power are added but of different powers remain as it is.

$9x^4+5x^3+ 2x$

$3x^4+4x^2+7x$

$12x^4+5x^3+4x^2+9x$

Multiplication of Polynomials:

In general, when multiplying two polynomials together, the distributive property is used, i.e.every term of one polynomial is multiplied with every term of the other polynomial. After that the answer is simplified by combining the like terms. In the following example every term of poly 2 will multiply with every term of poly 1. Coefficients get multiplied and power gets added.

Poly 1: $5x^3 + 3x^2+2$

Poly 2: $3x^2+5x+4$

After multiplying, the result is

$15x^5 + 9x^4 + 6x^2 + 25x^4 + 15x^3 + 10x + 20x^3 + 12x^2 + 8$

Simplifying the answer by adding the like terms,

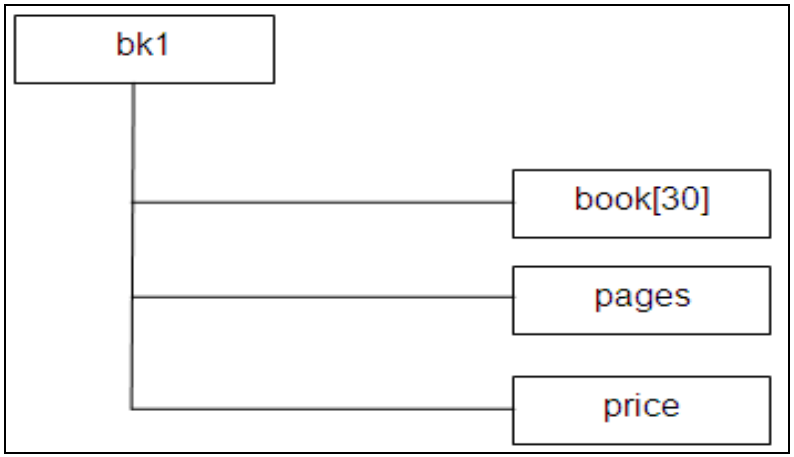
Multiplication result: $15x^5 + 34x^4 + 35x^3 + 18x^2 + 10x + 8$

1.7 Structure :

Arrays can store many values of a similar data type. Data in the array is of the same composition in nature as far as the type is concerned. To maintain employee's information one should have information such as name, age, qualification, salary and so on. Name and qualification of the employee are char data type, age is an int and salary is float. All these data types cannot be expressed in a single array. One may think to declare different arrays for each data type, but there will be huge increase in source codes of the program. Hence, arrays cannot be useful here. For tackling such a mixed data type problems, a special feature is provided by C known as a structure.

Example: A structure of type book 1 is created. It consists of three members: book [30] of char data type, pages of int type and price of float data type. Figure explains various members of a structure.

```
struct  book1
{
char book[30] ;
int pages;
float price;
} ;
struct book1 bk1;
```



Block Diagram of a Structure

1.8 Self-Referential Structure:

b) Declaration of Linked Structure:

The linked structure given in abovefigure can be obtained by the following steps:

Declare structure chain.

Declare variables A and B of type chain.

p(A) = B

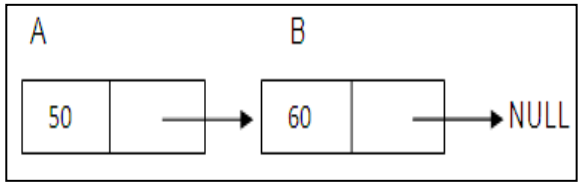
These steps have been coded in the program segment given below:

```
struct chain                    /* declare structure chain */
 {
        int val;
        chain *p:
};
        struct chain A, B;                      /* declare structure variables A and B
        A.p =  &B;              /* Connect  A to B
         B.p = NULL;
```

The data elements in this linked structure can be assigned as follows:

```
        A.val= 50;
        B.val =60;
```

The linked structure now looks like as shown in figure.



Value Assignment to Data Elements

Self-Referential Structure:

c) Advantages of Self-referential Structure:

The self-referential structures have three main advantages over arrays:

1) Flexibility for Memory Allocation:

It is not necessary to know the number of elements and allocate memory in advance for self-referential structures. Memory can be allocated as and when necessary. Insertion and deletion from self-referential structure is efficient using pointers. The individual elements can be scattered anywhere in memory and no contiguous memory is required like array elements.

2) Useful in Programming Constructs:

The self-referential structures are extensively used in programming constructs: linked lists, stacks, Queues and trees etc.

d) typedef keyword:

The C programming language provides a keyword called typedef, which is used to give a type a new name. typedef can be used to give a name to user defined data type. To use typedef with structure define a new data type and then use that data type to define structure variables directly.

Example: typedef unsigned char BYTE;

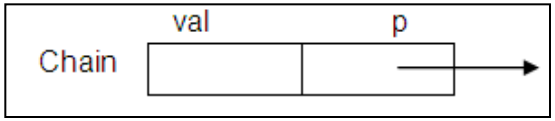
After this type definitions, the identifier BYTE can be used as an abbreviation for the type unsigned char,for example BYTE b1, b2;

1.8 Self-Referential Structure:

When a member of a structure is declared as a pointer to the structure itself then the structure is called self-referential structure.

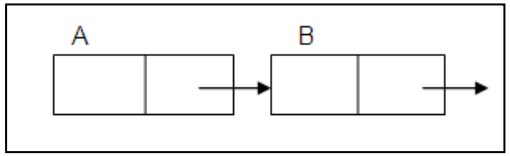
```
struct chain
{
        Intval;
        struct chain *p;
};
```

The structure called 'chain' consists of two members: val and p. The member val is a variable of type int whereas the member p is a pointer to a structure of type chain. Thus, the structure chainhas a member that can point to a structure of type chain or may be itself .This type of self-referencing structure can be viewed as shown in Figure.



Self –referential Structure Chain

Since pointer p can point to a structure variable of type chain, connecting the two such structure variables, A and B, linked structure is obtained as shown in below Figure.



Linked Structure

Unit 2 Linear Data Structure

2.1 Introduction to Array – array representation

2.2 Sorting algorithms with efficiency

- Bubble sort,
- Insertion sort,
- Merge sort,
- Quick Sort,
- Selection Sort

2.3 Searching techniques –

1. Linear Search
2. Binary search

2.1 Introduction to Array- array representation

- The fundamental data types namely int, float, char etc. are very useful but variable of these data type can be stored only one value at a time. So they can handle limited amount of data. In many applications we need to handle large volume of data for that we have to need powerful data types that would facilitate efficient storing, accessing and manipulation of data items.
- C supports derived data type known as Array.
- Definition-
- An array is collection of data items of the same data type
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- An array is also called as Subscripted Variables
-

Features of Array:

- An array is a collection of similar elements.
- The location of array is the location of its first element.
- The first element in array is numbered zero so the last element is less than the size of array.
- The length of array is the number of its elements in array.
- The type of an array is the data type of its element.
- An array is known as subscripted variable.
- Before using array it's type and dimension must be declared.

Single Dimension Array :

A list of items can be given one variable name using only one subscript and such a variable is called a single subscripted or single Dimension Array.

Syntax :

data type arrayname[size];

Example :

```
int rollno[3];
float marks[5];
char name[30];
```

Array elements are always stored in contiguous memory locations and since the data type int occupies 2 bytes of memory, each element will be allocated 2 bytes.

Declaration and Initialization Array :

We can initialise elements of array in the same way the ordinary variable.

Syntax :

data type arrayname[size]={list of values};

Example :

```
int rollno[5]={1,2,3,4,5};
int rollno[ ]={1,2,3,4,5};
float num[5]={2.5,7.2,9.2,6.2,3.3};
```

Multi Dimension Array :

An array whose elements are speacified by more than one subscript is known as multi dimension array (also called Matrix)

Syntax :

data type arrayname[row size][column size];

Example :

int student[5][2];

	C0	C1
R0	1	67
R1	2	73
R2	3	82
R3	4	90
R4	5	58

char name [4][10];

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9
R0	S	W	A	P	N	I	L	\0		
R1	S	A	N	T	O	S	H	\0		
R2	J	A	Y	D	I	P	\0			
R3	S	A	N	D	I	P	\0			

Declaration and Initialisation Array :

int number[3][4]={8,12,25,37,42,52,68,79,81,92,100,102};

char city[5][10]={“Mumbai”,”Punei”,”Satara”,”Kolhapur”,”Sangli”};

String (Character Array) :

In C character string simply treats as array character. The size in a character string represents the maximum number of characters that the string can hold.

Example :

char name[10];

It declares the name as a character array (string) variable that can hold a maximum 10 characters. Each character of the string is treated as an element of the array name and is stored in the memory as follows.

A Character string terminates with an additional null character. Thus the element in name[10] holds the null character '\0'. When declaring character arrays, we must allow one extra element space for the null terminator.

Declaration and Initialisation Array

We can initialise elements of array in the same way as the ordinary variable.

Syntax :

data type arrayname[size]={list of values};

Example :

```
char name[5]={'s','w','a','p','n','i','l','\0'};
char name[]="siddhi";
char name[5]={'P'};
char  *colour[]={"Red","Green","Blue","Yellow"};
```

2.2 Sorting Algorithms

What is Sorting?

Sorting is a process of ordering or placing a list of elements from a collection in some kind of order. It is nothing but storage of data in sorted order. Sorting can be done in ascending and descending order. It arranges the data in a sequence which makes searching easier.

For example, suppose we have a record of employee. It has following data:

Employee No.
Employee Name
Employee Salary
Department Name

Here, employee no. can be takes as key for sorting the records in ascending or descending order. Now, we have to search a Employee with employee no. 116, so we don't require to search the complete record, simply we can search between the Employees with employee no. 100 to 120.Sorting Techniques Sorting technique depends on the situation. It depends on two parameters.

1. Execution time of program that means time taken for execution of program.
2. Space that means space taken by the program.

Sorting techniques are differentiated by their efficiency and space requirements.

Sorting can be performed using several techniques or methods, as follows:

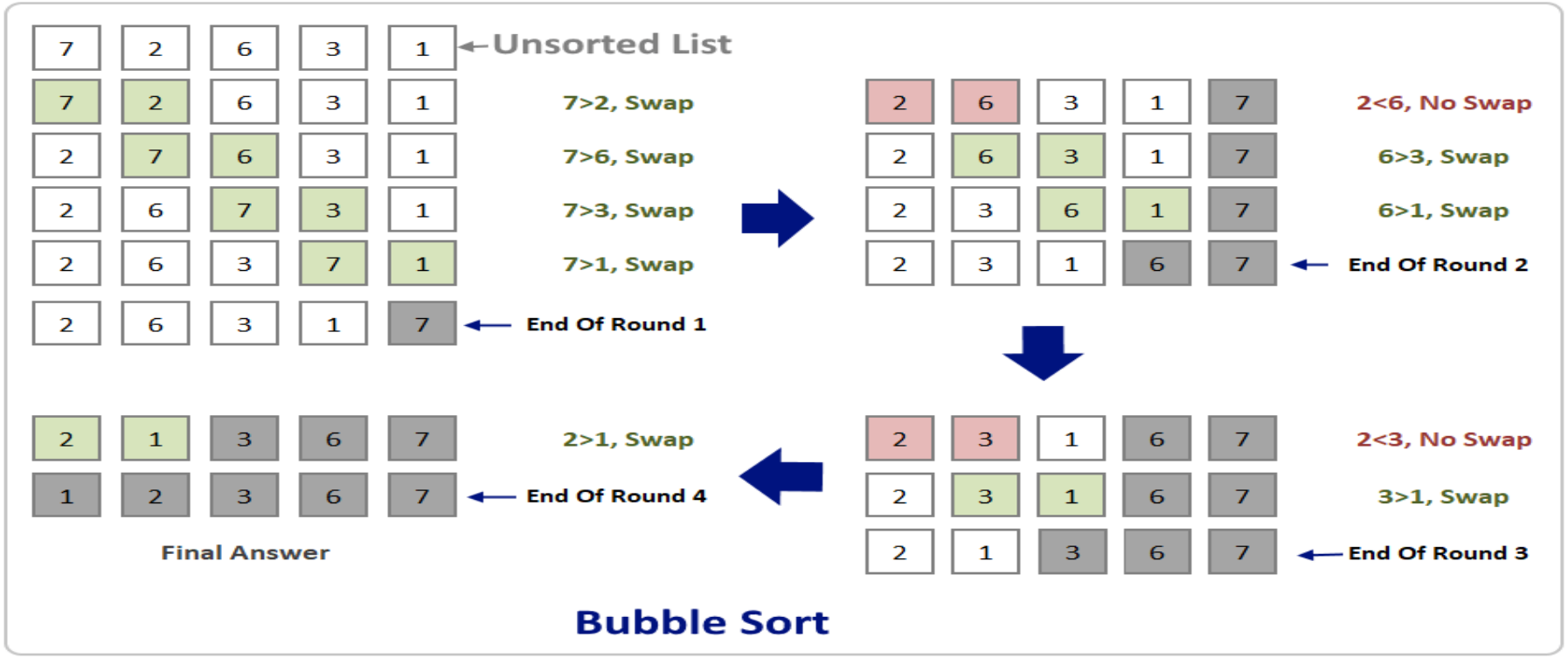
1. Bubble Sort
2. Insertion Sort
3. Merge Sort
4. Quick Sort
5. Selection Sort

1. Bubble Sort

Bubble sort is a type of sorting.

It is used for sorting 'n' (number of items) elements.

It compares all the elements one by one and sorts them based on their values.



2. Insertion Sort

- Insertion sort is a simple sorting algorithm.
- This sorting method sorts the array by shifting elements one by one.
- It builds the final sorted array one item at a time.
- Insertion sort has one of the simplest implementation.
- This sort is efficient for smaller data sets but it is insufficient for larger lists.
- It has less space complexity like bubble sort.
- It requires single additional memory space.
- Insertion sort does not change the relative order of elements with equal keys because it is stable.

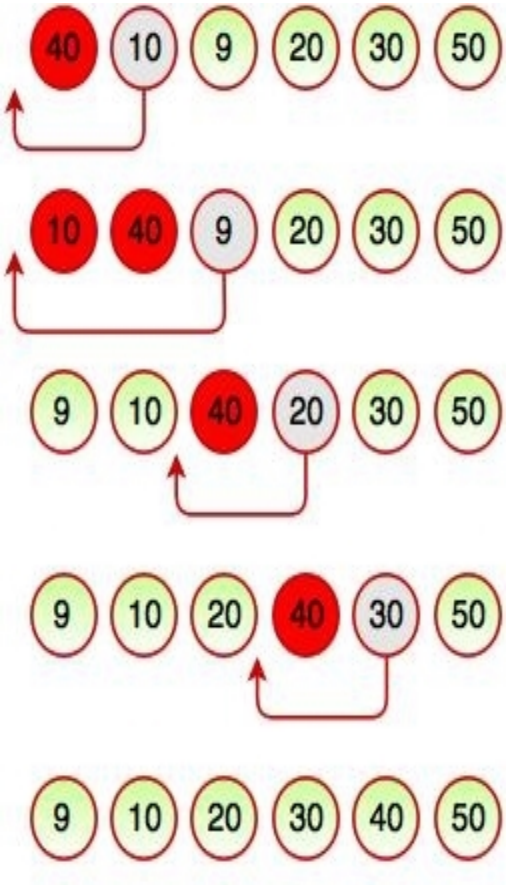


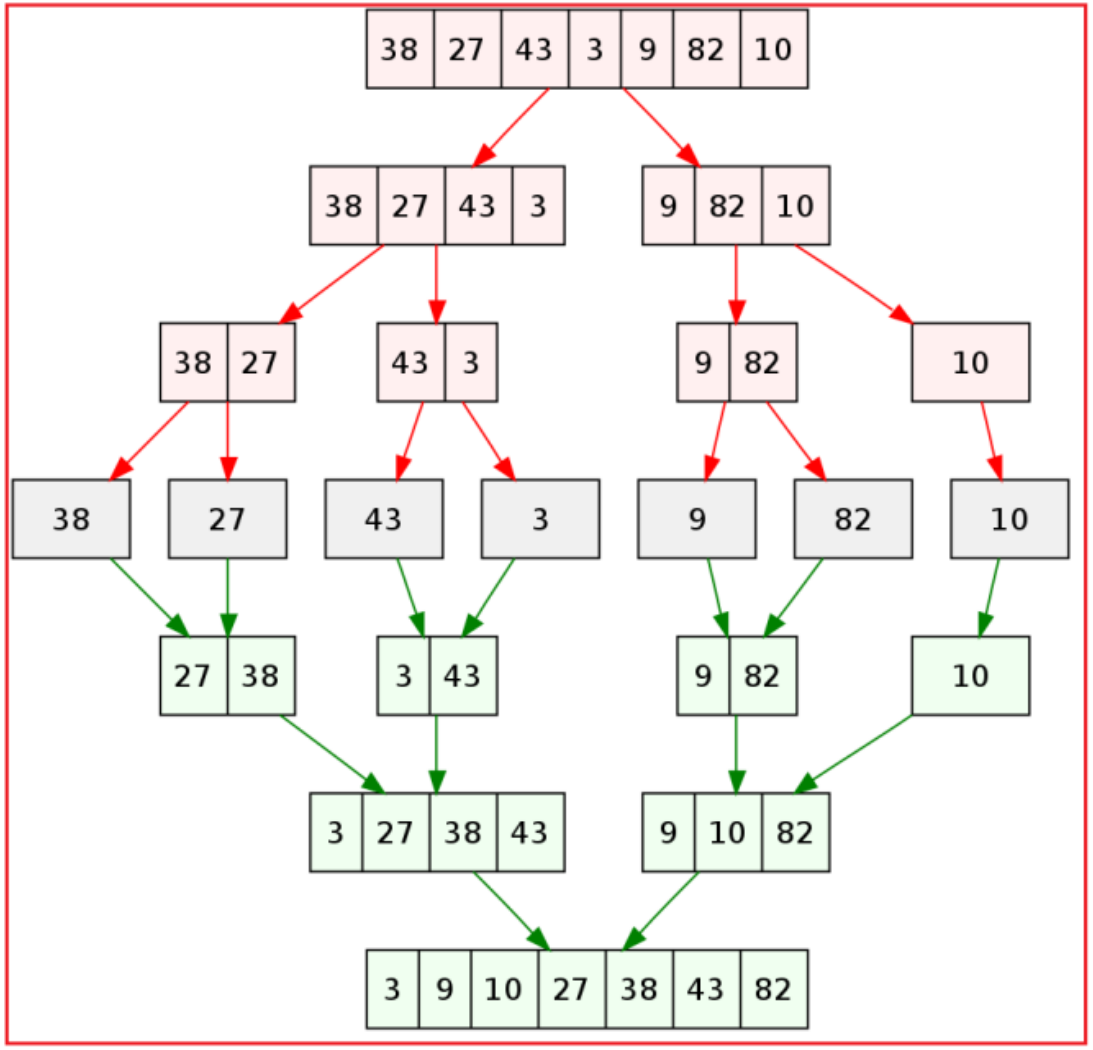
Fig. Working of Insertion Sort

The above diagram represents how insertion sort works. Insertion sort works like the way we sort playing cards in our hands. It always starts with the second element as key. The key is compared with the elements ahead of it and is put it in the right place.

In the above figure, 40 has nothing before it. Element 10 is compared to 40 and is inserted before 40. Element 9 is smaller than 40 and 10, so it is inserted before 10 and this operation continues until the array is sorted in ascending order.

3. Merge sort

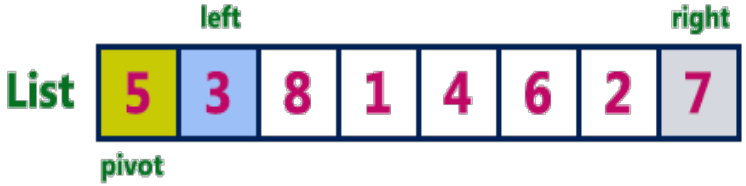
Merge sort is one of the most efficient sorting algorithms. It works on the principle of Divide and Conquer. Merge sort repeatedly breaks down a list into several sublists until each sublist consists of a single element and merging those sublists in a manner that results into a sorted list.



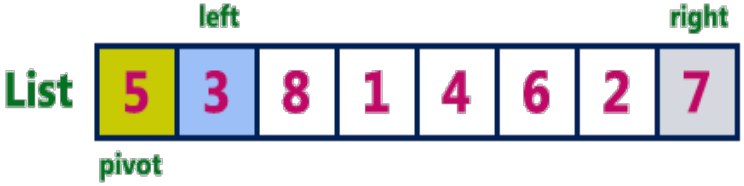
4. Quick Sort

Consider the following unsorted list of elements...

Define pivot, left & right. Set pivot = 0, left = 1 & right = 7. Here '7' indicates 'size-1'.



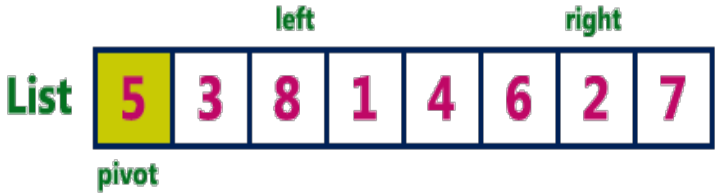
Compare List[left] with List[pivot]. If **List[left]** is greater than **List[pivot]** then stop left otherwise move left to the next.
Compare List[right] with List[pivot]. If List[right] is smaller than List[pivot] then stop right otherwise move right to the previous.
Repeat the same until **left>=right**.
If both left & right are stoped but left<right then swap List[left] with List[right] and countinue the process.
If left>=right then swap List[pivot] with List[right].



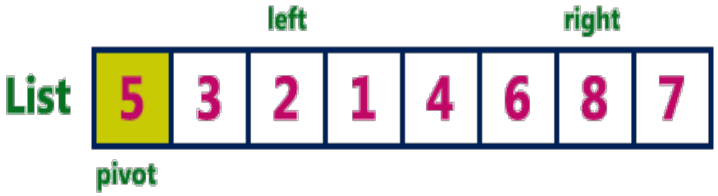
Compare List[left]<List[pivot] as it is true increment left by one and repeat the same. left will stop at 8

Compare List[left]<List[pivot] as it is true increment left by one and repeat the same, left will stop at 8.

Compare List[right]>List[pivot] as it is true decrement right by one and repeat the same, right will stop at 2.



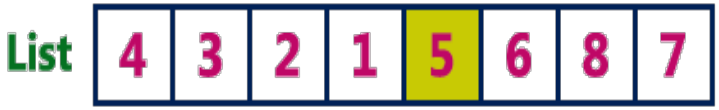
Here left & right both are stoped and left is not greater than right so we need to swap List[left] and List[right]



Compare List[left]<List[pivot] as it is true increment left by one and repeat the same, left will stop at 6.

Compare List[right]>List[pivot] as it is true decrement right by one and repeat the same, right will stop at 4.

Here left & right both are stoped and left is greater than right so we need to swap List[pivot] and List[right]



Here we can observe that all the numbers to the left side of 5 are smaller and right side are greater. That means 5 is placed in its correct position.

Repeat the same process on the left sublist and right sublist to the number 5.

In the left sublist as there are no smaller number than the pivot left will keep on moving to the next and stops at last number. As the List[right] is smaller, right stops at same position. Now left and right both are equal so we swap pivot with right.

In the right sublist left is grester than the pivot, left will stop at same position.

As the List[right] is greater than List[pivot], right moves towords left and stops at pivot number position.

Now left > right so we swap pivot with right. (6 is swap by itself).

Repeat the same recursively on both left and right sublists until all the numbers are sorted.

The final sorted list will be as follows...

5. Selection Sort

Selection sort is a simple sorting algorithm which finds the smallest element in the array and exchanges it with the element in the first position. Then finds the second smallest element and exchanges it with the element in the second position and continues until the entire array is sorted.

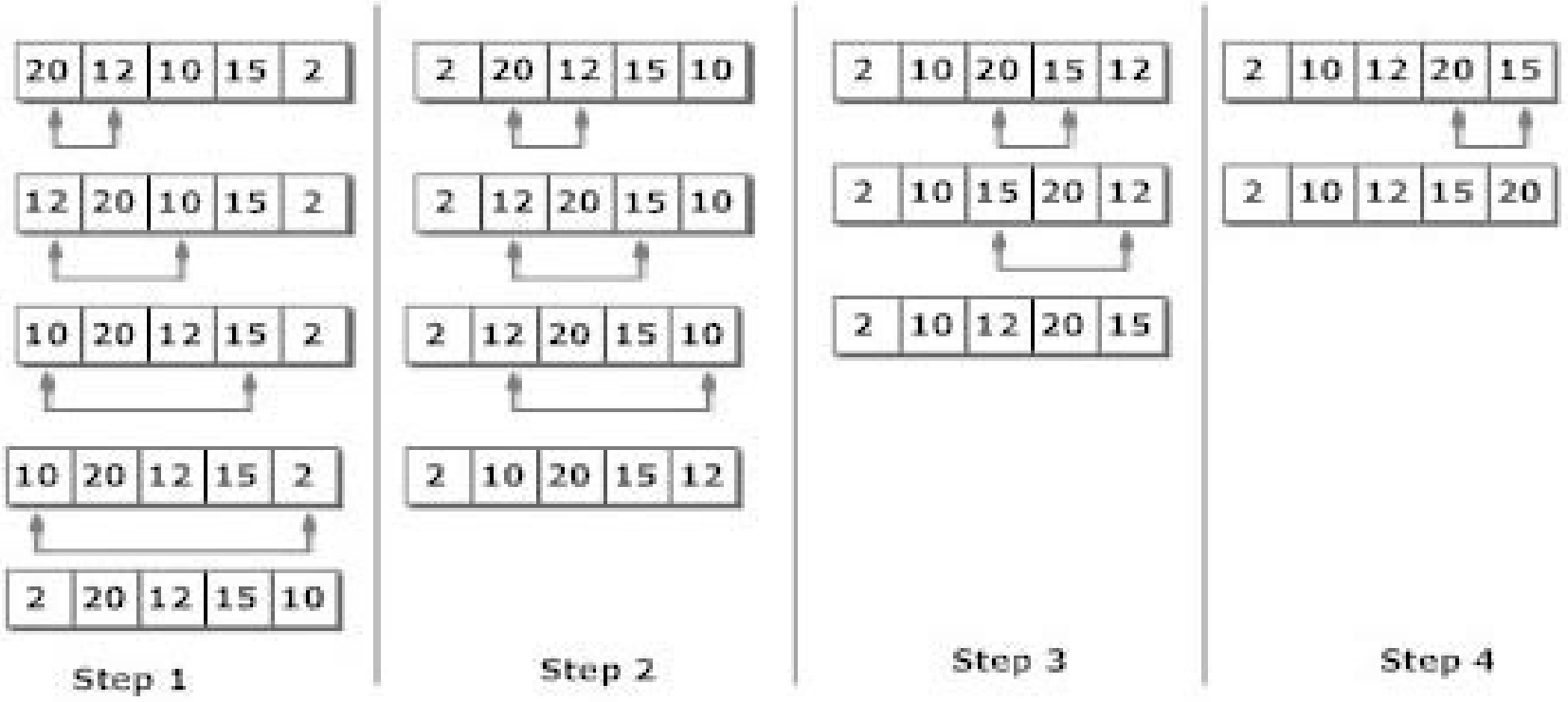


Figure: Selection Sort

2.3 Searching Techniques (Linear and Binary Search)

What is Searching?

- Searching is the process of finding a given value position in a list of values.
- It decides whether a search key is present in the data or not.
- It is the algorithmic process of finding a particular item in a collection of items.
- It can be done on internal data structure or on external data structure.

Searching Techniques

- **To search an element in a given array, it can be done in following ways:**

1. Linear/sequential Search
2. Binary Search

1. Sequential Search

- Sequential search is also called as Linear Search.
- Sequential search starts at the beginning of the list and checks every element of the list.
- It is a basic and simple search algorithm.
- Sequential search compares the element with all the other elements given in the list. If the element is matched, it returns the value index, else it returns -1.

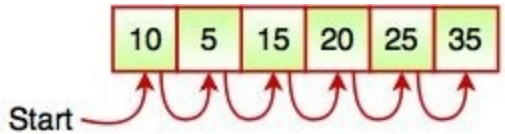


Fig. Sequential Search

The above figure shows how sequential search works. It searches an element or value from an array till the desired element or value is not found. If we search the element 25, it will go step by step in a sequence order. It searches in a sequence order. Sequential search is applied on the unsorted or unordered list when there are fewer elements in a list.

2. Binary Search

- Binary Search is used for searching an element in a sorted array.
- It is a fast search algorithm
- Binary search works on the principle of divide and conquer.
- This searching technique looks for a particular element by comparing the middle most element of the collection.
- It is useful when there are large number of elements in an array.
-

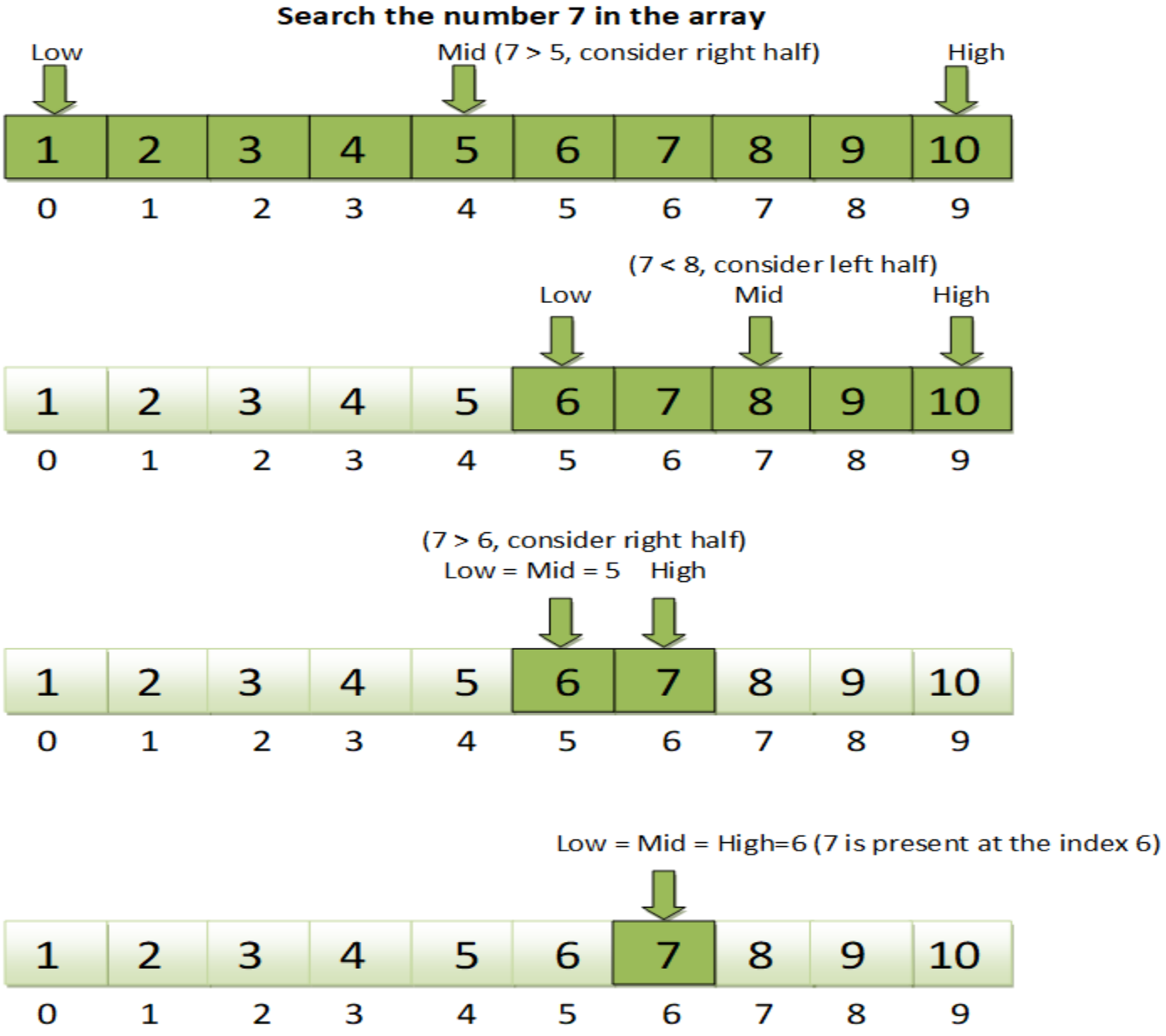
The above array is sorted in ascending order. As we know binary search is applied on sorted lists only for fast searching.

- **For example,** if searching an element 25 in the 7-element array, following figure shows how binary search works:

Fig. Working Structure of Binary Search

Binary searching starts with middle element. If the element is equal to the element that we are searching then return true. If the element is less than then move to the right of the list or if the element is greater than then move to the right of the list. Repeat this, till you find an element.

Binary Search



3. Linked List

3.1 Introduction to Linked List

3.2 Implementation of Linked List – Static & Dynamic representation,

3.3 Types of Linked List

- Singly Linked list(All type of operation)

- Doubly Linked list (Create , Display)

- Circularly Singly Linked list (Create, Display)

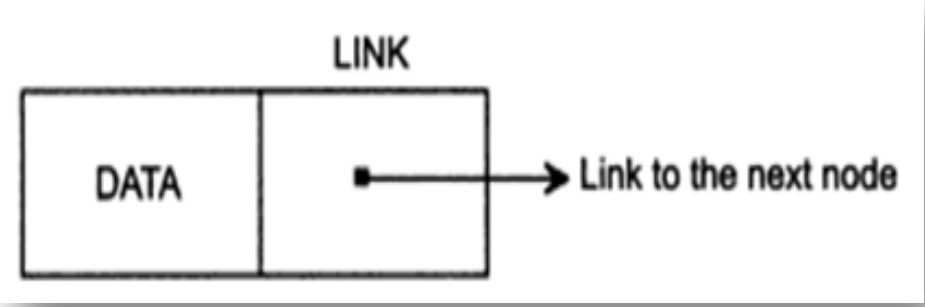
- Circularly Doubly Linked list (Create, Display)

3.4 Generalized linked list – Concept and Representation

3.1 Introduction to Linked List

- Linked list is a linear dynamic data structure. It is a collection of some nodes containing homogeneous elements.
- Each node consists of a data part and one or more address part depending upon the types of the linked list.
- There three different types of linked list available which are

1. Singly linked list
2. Doubly linked list
3. Circular linked list



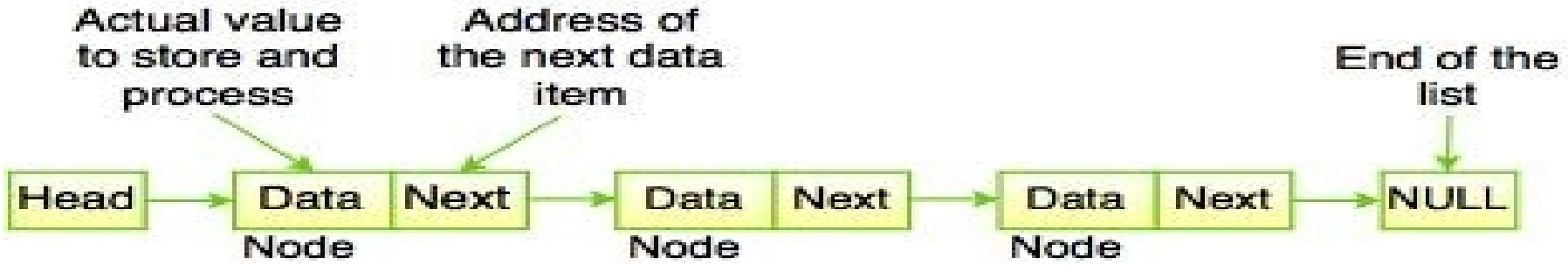


Fig. Linked List

Advantages of Linked Lists:

1) **Facilitate Dynamic Memory Management:**

Linked lists facilitate dynamic memory management by allowing elements to be added or deleted at any time during program execution.

2) **Ensures Efficient Utilization of Memory Space:**

The use of linked lists ensures efficient utilization of memory space as only that much amount of memory space is reserved as is required for storing the list elements.

3) **Easy to Manipulate:**

It is easy to insert or delete elements in a linked list, unlike arrays, which require shuffling of other elements with each insert and delete operation.

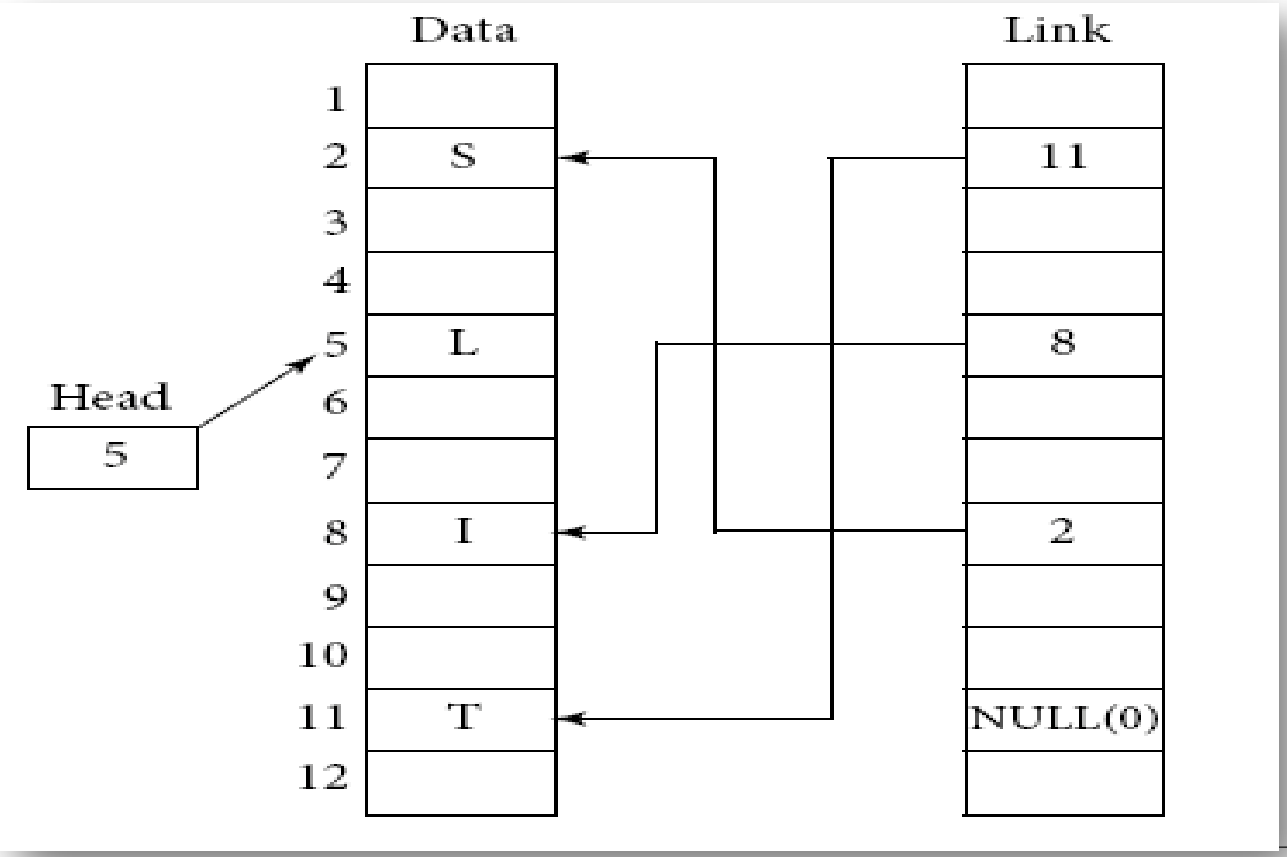
Difference between Linked List and Array:

	Linked List	Array
1)	The linked list is a collection of nodes and each node is having one data field and next data field.	The array is a collection of similar types of data elements. In arrays the data is always stored at some index of array.
2)	Any element can be accessed by sequential access only.	Any element can be accessed by randomly i.e. with the help of index of array.
3)	Physically data can be deleted	Only logical deletion of data is possible.
4)	Insertion and deletion of data is easy.	Insertion and deletion of data is easy.
5)	Memory allocation is dynamic. Hence developer can allocate as well as deallocate the memory, so no wastage of memory is there.	The memory allocation is static. Hence once the fixed amount of size is declared then that much memory is allocated. So there is a chance of memory wastage or memory shortage.

3.2 Implementation of linked List-Static and Dynamic Representation

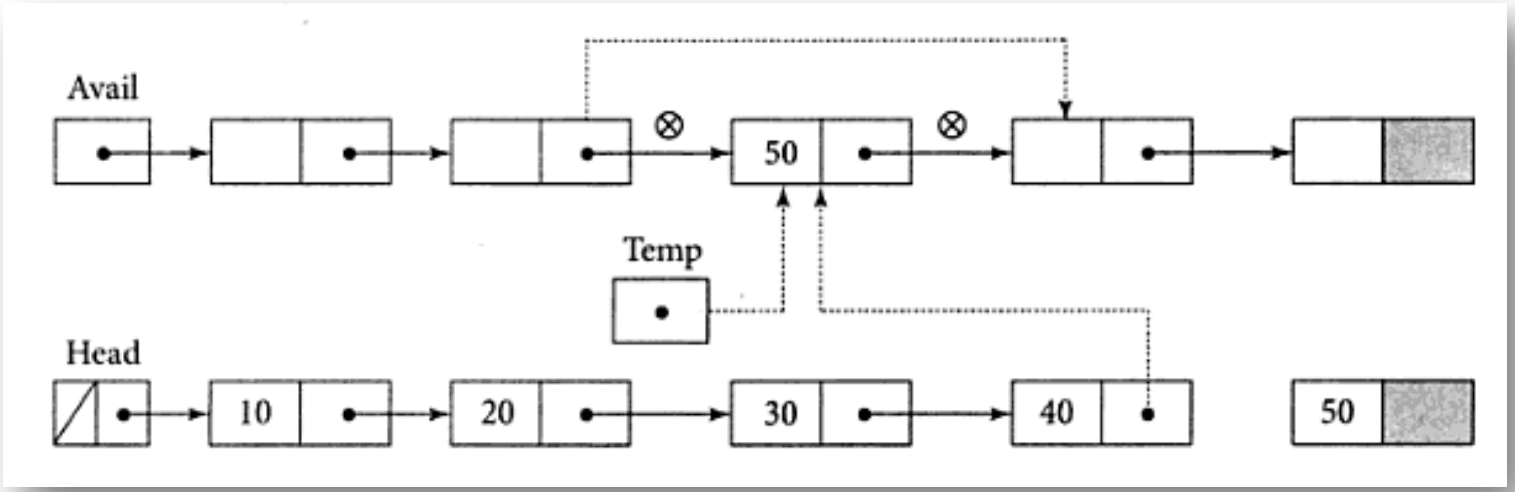
A) Static Representation:

The linked list is maintained by two linear arrays- one is used for data and the other for links. Let DATA and LINK be the two arrays, DATAcontains the information part, and their corresponding pointers to the next node are stored in the array LINK.

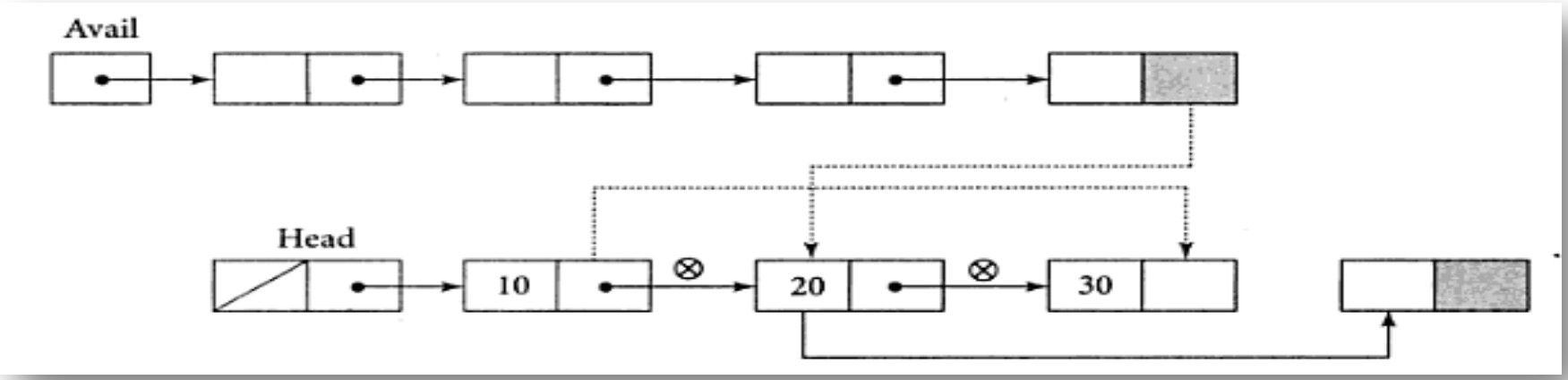


B) Dynamic Representation:

In this representation, a memory bank that is a collection of free memory space and memory manager program is used.



- In above Figure , a new node is taken from the AVAIL and temporarily holds the address of the new node in the variable.
- The new node is then added to the existing list. The dotted arrow shows the insertion of the new node to the list and the symbol is used to represent the deletion of links.
- The head node in the list does not contain any data.



In above figure, deletions of the new node from the existing list are done and it is returned to the memory bank i.e. AVAIL. The LINK field of the last node in the AVAIL will be pointing to the deleted node.

3.3 Types of Linked List

1. Singly Linked List

- Singly linked list is a linked list which is a linear list of some nodes containing homogeneous elements .
- Each node in a singly linked list consists of two parts,

1. One is data part

The data part contains the data or information and except the last node.

2. Address part.

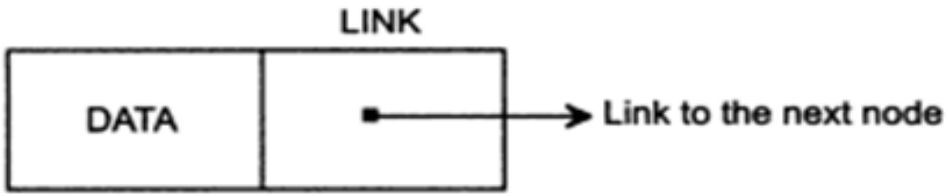
The address part contains the address of the next node in the list.

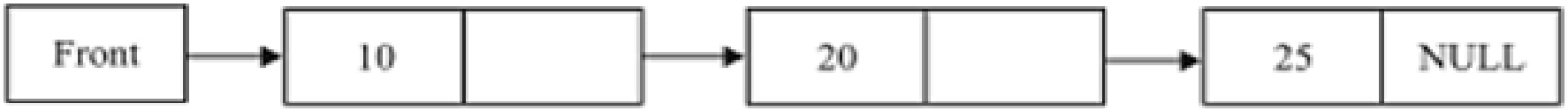
The address part of the last node in the list contains NULL.

Here one pointer is used to point the first node in the list.

Three basic operations on singly linked list which are

1. Insertion of a new node,
2. Deletion of a node
3. Traversing the linked list.



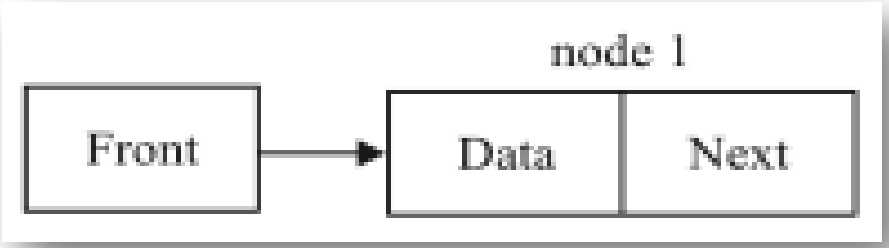


A) Operations on a Singly Linked List:

1) Creating a Singly Linked List:

A node of a linked list is a structure because it contains data of different types. In addition to the

information part, it contains a pointer that can point to a node i.e. to itself or to some other node

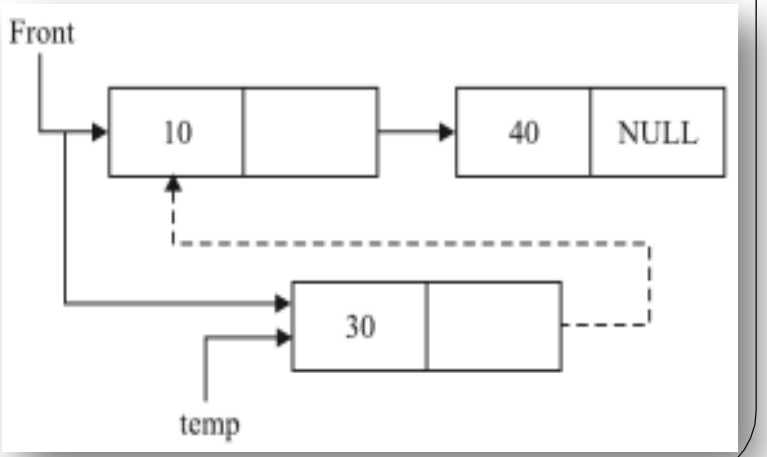


2) Insertion of a Node into in a Singly Linked List:

a) Inserting a Node at the beginning of the List:

Let the linked list be pointed by the front pointer.

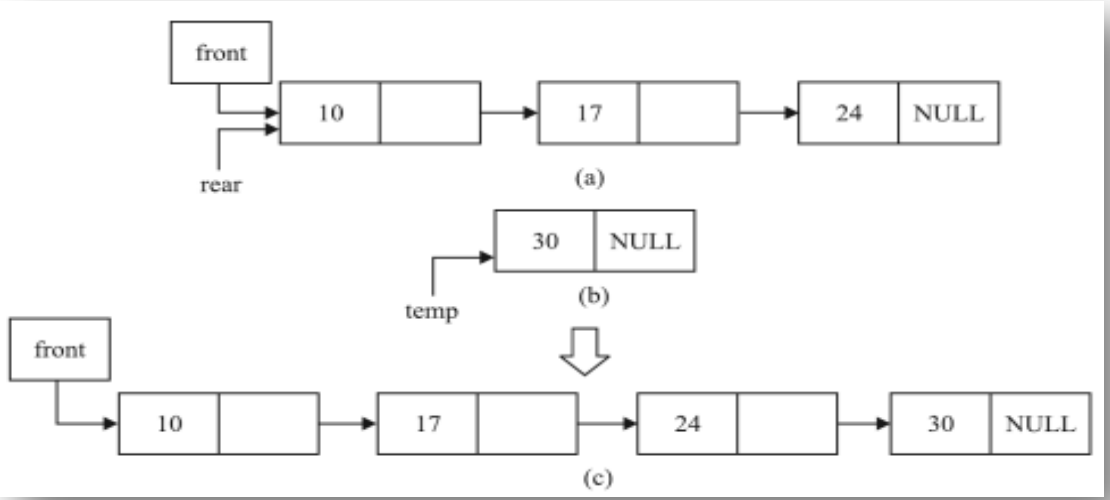
Let information to be inserted be val1.



b) Inserting Node at the End of Linked List:

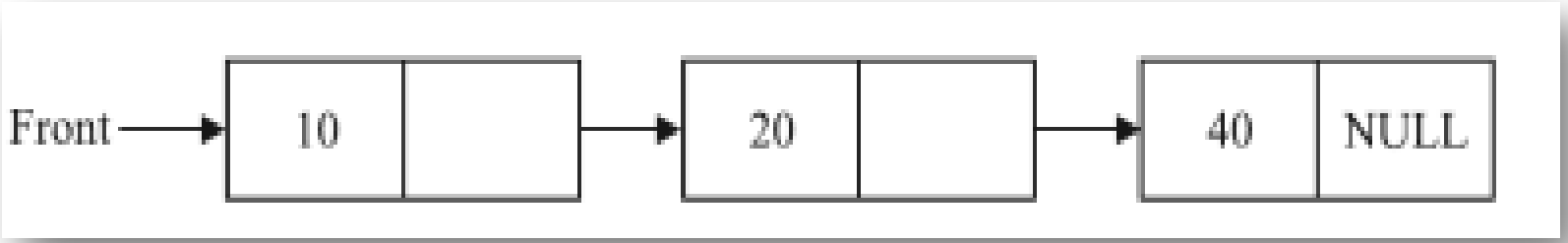
Let the linked list be pointed by front. Let information to be inserted in value field be 30.

Let another pointer ‘rear’ point to where front points.

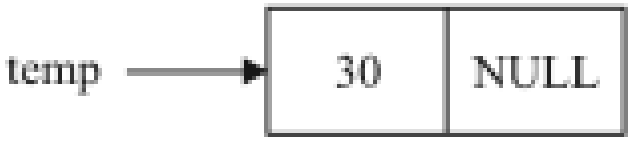


c) Inserting a Node at Specific Position in a Singly Linked List:

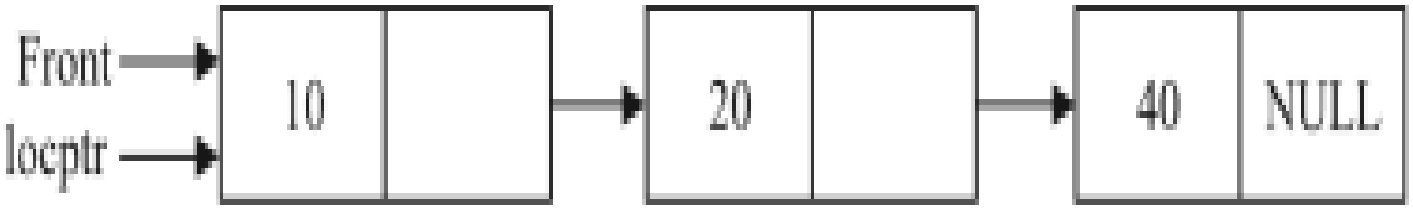
Consider a linked list containing 3 nodes as shown in Figure.



Let the node to be inserted is temp as shown in below figure 4.8 (d) after the 2nd node in the list. This means there should be some form of counter to count the number of nodes so as to reach at the desired position.



Node to be Appended

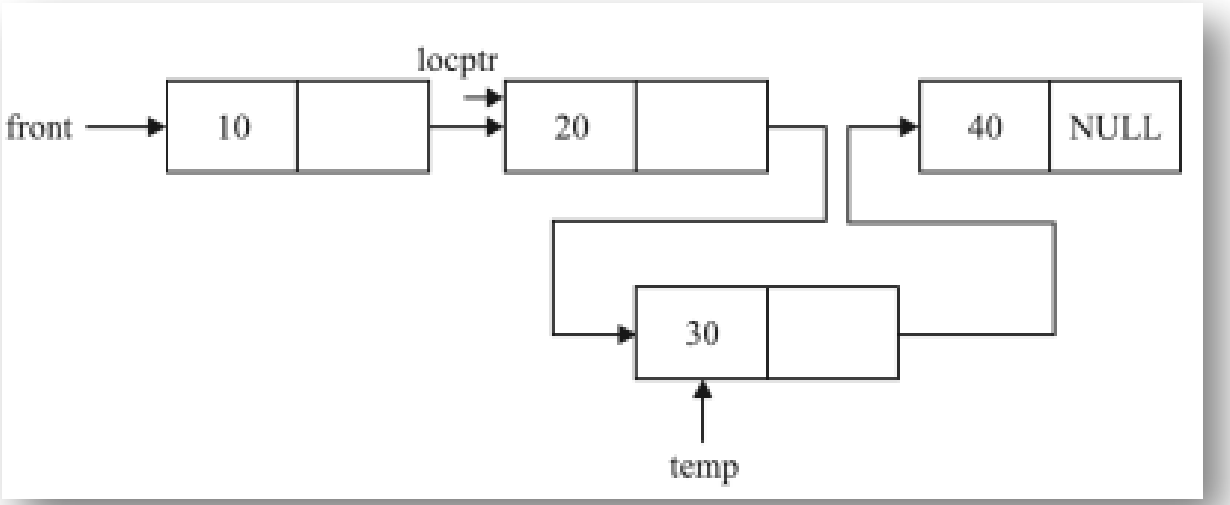


Original Linked List with a New Pointer

The node to be inserted must appear after the second node. Now; the next pointer of the node 30 should be made point to node 40. The next pointer of the node, where temp is currently positioned must be made to point, where pointer temp points. This is done by the following two lines of code.

```
temp->next=locptr->next;
locptr->next = temp;
```

Temp points to the node 30 that is to be appended. This is shown in below Figure



Linked list after appending a node

3) Traversing Singly Linked List:

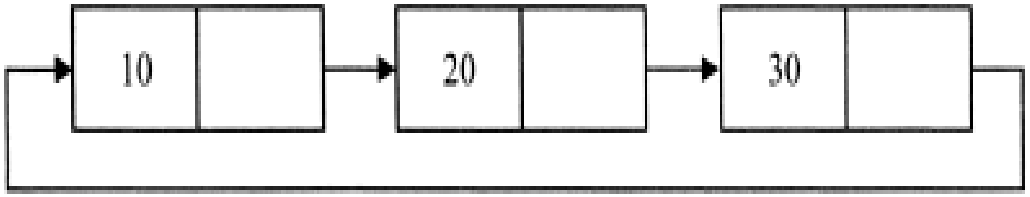
Traversing linked list means visiting each and every node of the singly linked list. For traversing the singly linked list firstly move to the first node, fetch the data from the node and perform the operations such as arithmetic operation or any operation depending on data type.

2) Doubly Linked List:

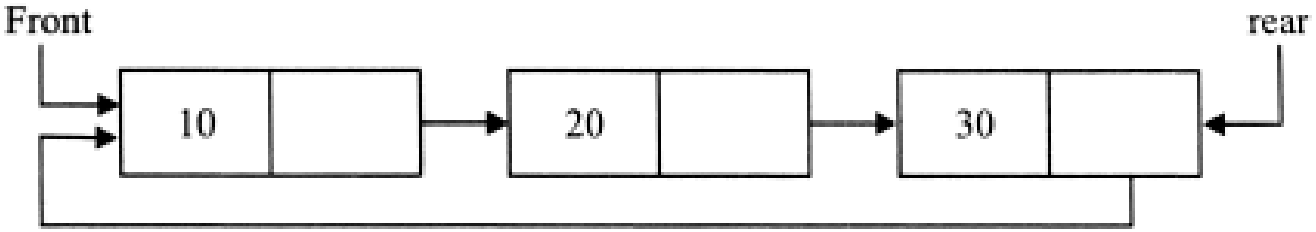
- Each node of the doubly linked list has two pointer fields and holds the address of predecessor and successor elements.
- These pointers enable bi-directional traversing, i.e. traversing the list in backward and forward direction.
- In several applications, it is very essential to traverse the list in backward direction. The pointer pointing to the predecessor node is called left link and pointer pointing to successor is called right link.
- The pointer field of the first and last node holds NULL value, i.e. the beginning and end of the list can be identified by NULL value. The structure of the node is as shown in Figure

3) Circularly Linked List:

A circular list is one in which the next field of the last node points to the first node of the list or the next field of the last node contains the address of the first node of the list.

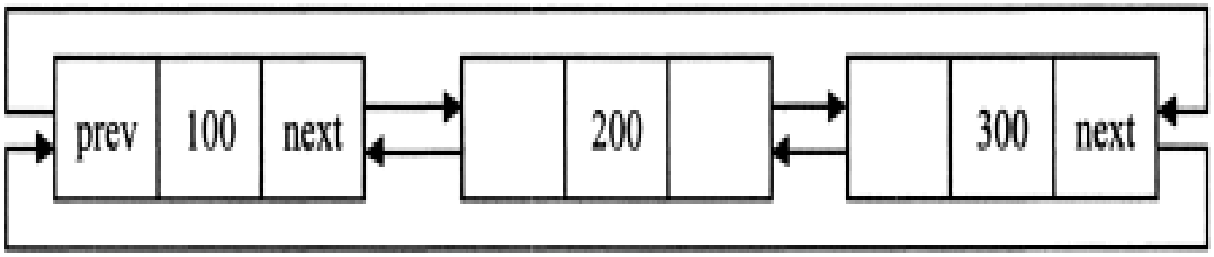


The two pointers front and rear can be used to associate with the first and last node respectively.



B) Circularly Doubly Linked List:

A circularly double linked list is one in which the next field of last node points the first node and the previous field of the first node points the last node.



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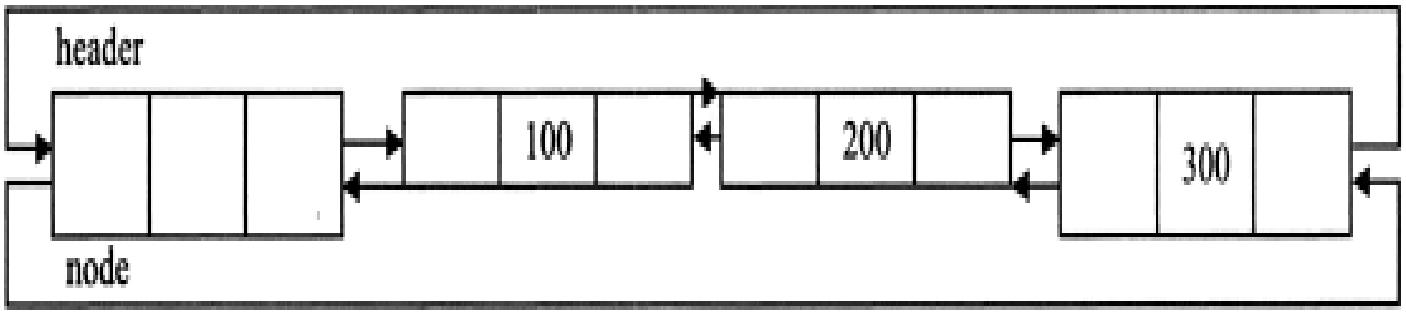


of the list.

UNIT 4: Stack

4.1 Introduction

4.2 Representation of Stacks

4.3 Primitive Operations on Stacks

4.4 Applications of Stacks

4.5 Conversion of Infix, Prefix,Postfix

Evaluation of Postfix and Prefix

1.1 Introduction

What is Stack?

- Stack is an ordered list of the same type of elements.
- It is a linear list where all insertions and deletions are permitted only at one end of the list.
- Stack is a LIFO (Last In First Out) structure.
- In a stack, when an element is added, it goes to the top of the stack.

Definition

"Stack is a collection of similar data items in which both insertion and deletion operations are performed based on LIFO principle".

Primitive Operations on Stack

1. **CREATE**
2. **PUSH**
3. **POP**
4. **ISEMPTY**
5. **ISFULL**
6. **DISPLAY/TRAVERSE**

Primitive Operation on Stack

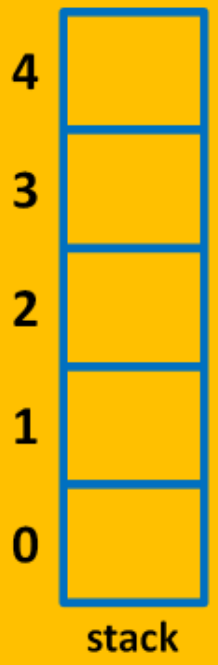
1. Create

   This operation create a stack, which is empty

Operations on STACK ?

- **Creation**
- Insertion
- Deletion
- Displaying

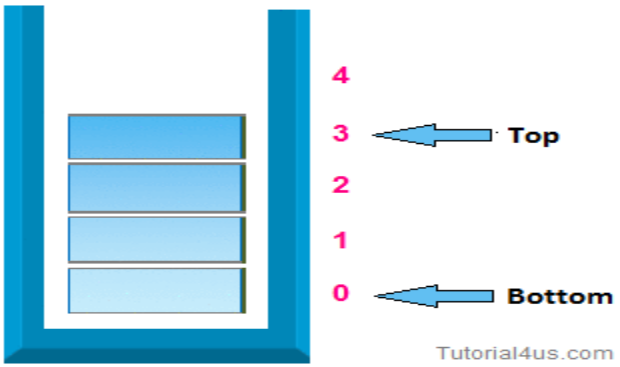
```
#define  SIZE  5
int stack[SIZE];
```



Operation on Stack

2. Push

The push operation adds a new element to the stack. As stated above, any element added to the stack goes at the top, so push adds an element at the top of a stack



push(value) - Inserting value into the stack

In a stack, push() is a function used to insert an element into the stack. In a stack, the new element is always inserted at **top** position. Push function takes one integer value as parameter and inserts that value into the stack. We can use the following steps to push an element on to the stack…

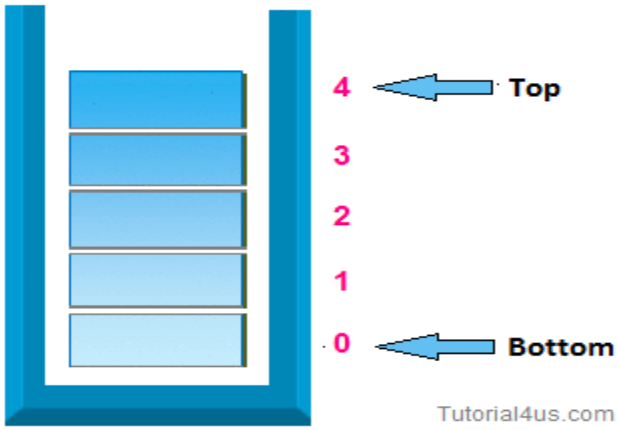
Step 1 - Check whether **stack** is **FULL**. (**top == SIZE-1**)

Step 2 - If it is **FULL**, then display **"Stack is FULL!!! Insertion is not possible!!!"** and terminate the function.

Step 3 - If it is **NOT FULL**, then increment **top** value by one (**top++**) and set stack[top] to value (**stack[top] = value**).

3 Pop

The pop operation removes and also returns the top-most (or most recent element) from the stack.



pop() - Delete a value from the Stack

In a stack, pop() is a function used to delete an element from the stack. In a stack, the element is always deleted from **top** position. Pop function does not take any value as parameter. We can use the following steps to pop an element from the stack...

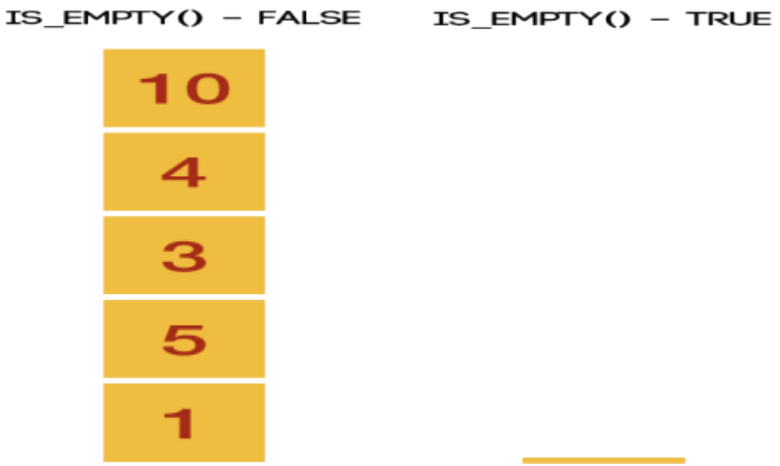
Step 1 - Check whether **stack** is **EMPTY**. (**top == -1**)

Step 2 - If it is **EMPTY**, then display **"Stack is EMPTY!!! Deletion is not possible!!!"** and terminate the function.

Step 3 - If it is **NOT EMPTY**, then delete **stack[top]** and decrement **top** value by one (**top--**).

3. isEmpty()

- This operation checks whether a stack is empty or not i.e., if there is any element present in the stack or not.
- When a stack is completely full, it is said to be Overflow state and if stack is completely empty, it is said to be Underflow state.



4. Isfull()

This operation checks whether the stack isfull. It returns TRUE if stack is full and false otherwise

5. display() - Displays the elements of a Stack

We can use the following steps to display the elements of a stack...

Step 1 - Check whether **stack** is **EMPTY**. (**top == -1**)

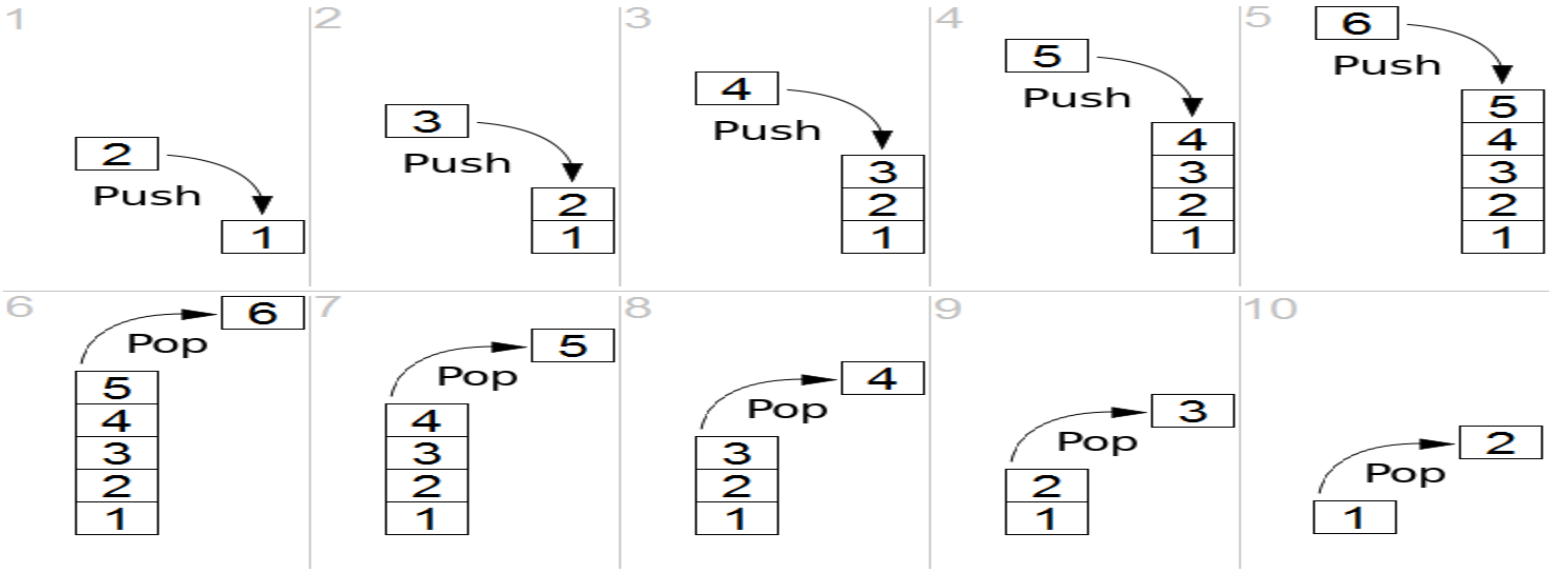
Step 2 - If it is **EMPTY**, then display "**Stack is EMPTY!!!**" and terminate the function.

Step 3 - If it is **NOT EMPTY**, then define a variable '**i**' and initialize with top. Display **stack[i]** value and decrement **i** value by one (**i--**).

Step 4 - Repeat above step until **i** value becomes '0'.

Representation / Implementation of Stack (Static Implementaton)

- The below diagram represents a stack insertion and deletion operation.
- In a stack, inserting and deleting of elements is performed at a single position which is known as, **Top**.
- Insertion operation can be performed using Push() function and deletion operation can be performed using Pop() function .
- New element is added and deleted at top of the stack
- Delete operation is based on LIFO principle.



Following table shows the Position of Top which indicates the status of stack

Position of Top	Status of Stack
-1	Stack is empty.
0	Only one element in a stack.
N - 1	Stack is full.
N	Stack is overflow. (Overflow state)

4.4 Applications of Stack :

Following are some of the important applications of a Stack data structure:

1. Stacks can be used for expression evaluation.
2. Stacks can be used to check parenthesis matching /correctness of nested parenthesis.
3. Reversing a string.
4. Check whether string is palindrome or not.
5. Stacks can be used for Conversion from one form of expression to another.

   A. Infix to Postfix or Infix to Prefix Conversion

   The stack can be used to convert some infix expression into its postfix equivalent, or prefix equivalent.

   B. Postfix or Prefix Evaluation

   These postfix or prefix notations are used in computers to express some expressions.
6. Stacks can be used for Memory Management.
7. Stack data structures are used in backtracking problems.

   Backtracking *can be defined as a general algorithmic technique that considers searching every possible combination in order to solve a computational problem.*

   backtracking is, that is solving all sub-problems one by one in order to reach the best possible solution.

   Decision Problem – In this, we search for a feasible solution.
   Optimization Problem – In this, we search for the best solution.
   Enumeration Problem – In this, we find all feasible solutions.

Functions of Stack

Push()	Pop()	Display()

void push()
{
int val;
if(top==MAX-1)
{
printf("\nStack is full!!");
}
else
{
printf("\nEnter element to push:");
scanf("%d",&val);
top=top+1;
stack[top]=val;
}
}
	void pop()
	{
	if(top==-1)
	{
	printf("\nStack is empty!!");
	}
	else
	{
	printf("\nDeleted element is %d",stack[top]);
	top=top-1;
	}
	}
		void pop()
		{
		int i;
		if(top == -1)
		{
		printf("\n\n Stack is Empty.");
		}
		else
		{
		for(i=top; i>=0; i--)
		{
		printf("\n%d", stack[i]);
		}
		}
		}

Prog. Stack using Array

```
#include <stdio.h>
#include <conio.h>
#define MAX 50
void push();
void pop();
void display();
int stack[MAX], top=-1, element;
void main()
{
   int ch;
   do
   {
      printf(" 1. push\n 2. popn 3. Display\n 4. Exit\n");
      printf("\n Enter Your Choice: ");
      scanf("%d", &ch);
      switch(ch)
      {
         case 1:
         push();
         break;
         case 2:
         pop();
         break;
         case 3:
         display();
         break;
         case 4:
         exit(0);
         default:
         printf("\n\n Invalid entry try again...\n");
      }
   }while(ch!=4);
   getch(); }
```

```
void push()
{
   if(top == MAX-1)
      printf("\n\n Stack is Full.");
   else
   {
      printf("\n\n Enter Element: ");
      scanf("%d", &element);
      top++;
      stack[top] = element;
  printf("\n\n Element Inserted = %d", element);
   }
}
void pop()
{
   if(top == -1)
      printf("\n\n Stack is Empty.");
   else
   {
      element = stack[top];
      top--;
   printf("\n\n Element Deleted = %d", element);
   }
}
void display()
{
   int i;
   if(top == -1)
   printf("\n\n Stack is Empty.");
   else
   {
      for(i=top; i>=0; i--)
         printf("\n%d", stack[i]);
   }
}
```

1.push/Insert

```
1. Insert
2. Delete
3. Display
4. Exit

Enter Your Choice: 1


Enter Element: 30


Element Inserted = 30



1. Insert
2. Delete
3. Display
4. Exit

Enter Your Choice:
```

2.Display

```
1. Insert
2. Delete
3. Display
4. Exit

Enter Your Choice: 3


30

20

10
```

3.pop/Delete

```
1. Insert
2. Delete
3. Display
4. Exit

Enter Your Choice: 2


Element Deleted = 30



1. Insert
2. Delete
3. Display
4. Exit

Enter Your Choice: 3


20

10
```

ARRAY VERSUS STACK

ARRAY	STACK
A data structure consisting of a collection of elements each identified by the array index	An abstract data type that serves as a collection of elements with two principal operations: push and pop
Contains elements of the same data type	Contains elements of different data types
Basic operations include insert, delete, modify, traverse, sort, search and merge	Basic operations are push, pop and peek
Any element can be accessed using the array index	Only the topmost element can be read or removed at a time

4.5 Expression Evaluation and Conversion

Infix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator in between the two operands.

Example: (A + B) * (C – D)

Prefix: It is the form of an arithmetic notation in which we fix (place) the arithmetic operator before (pre) its two operands. The prefix notation is called as polish notation (due to the polish mathematician Jan Lukasiewicz in the year 1920).

Example: * + A B – C D

Postfix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator after (post) its two operands. The postfix notation is called as suffix notation and is also referred to reverse polish notation.

Example: A B + C D – *

Infix	Prefix	Postfix
a + b	+ b a	a b +
(a + b) * (c + d)	* + d c + b a	a b + c d + *
b * b - 4 * a * c	- * c * a 4 * b b	b b * 4 a * c * -

Converting expressions using Stack:

Let us convert the expressions from one type to another. These can be done as follows:

1. Infix to postfix
2. Infix to prefix
3. Postfix to infix
4. Postfix to prefix
5. Prefix to infix
6. Prefix to postfix

Conversion from infix to postfix:

Procedure to convert from infix expression to postfix expression is as follows:

1. 1. Scan the infix expression from left to right.
2. If the scanned symbol is left parenthesis, push it onto the stack.
3. If the scanned symbol is an operand, then place directly in the postfix expression (output).
4. If the symbol scanned is a right parenthesis, then go on popping all the items from the stack and place them in the postfix expression till we get the matching left parenthesis.
5. If the scanned symbol is an operator, then go on removing all the operators from the stack and place them in the postfix expression, if and only if the precedence of the operator which is on the top of the stack is greater than (or greater than or equal) to the precedence of the scanned operator and push the scanned operator onto the stack otherwise, push the scanned operator onto the stack.

1. Infix to Postfix Conversion

Following table shows the evaluation of Infix to Postfix:

Example: Suppose we are converting A*B/(C-D)+E*F expression into postfix form.

Expression	Stack	Output
A	Empty	A
*	*	A
B	*	AB
/	/	AB*
(	/(	AB*
C	/(	AB*C
-	/(-	AB*C
D	/(-	AB*CD
)	-	AB*CD-
+	+	AB*CD-/
E	+	AB*CD-/E
*	+*	AB*CD-/E
F	+*	AB*CD-/EF
	Empty	**AB*CD-/EF*+**

So, the Postfix Expression is AB***CD-/EF*+**

Conversion of Infix to Postfix

Example to Convert Infix to Postfix using stack

a + (b*c)

Read character	Stack	Output
a	Empty	a
+	+	a
(	+(	a
b	+(	ab
*	+(*	ab
c	+(*	abc
)	+	abc*
		abc*+

EXAMPLE to convert infix expression to postfix

A+(B*C-(D/E-F)*G)*H

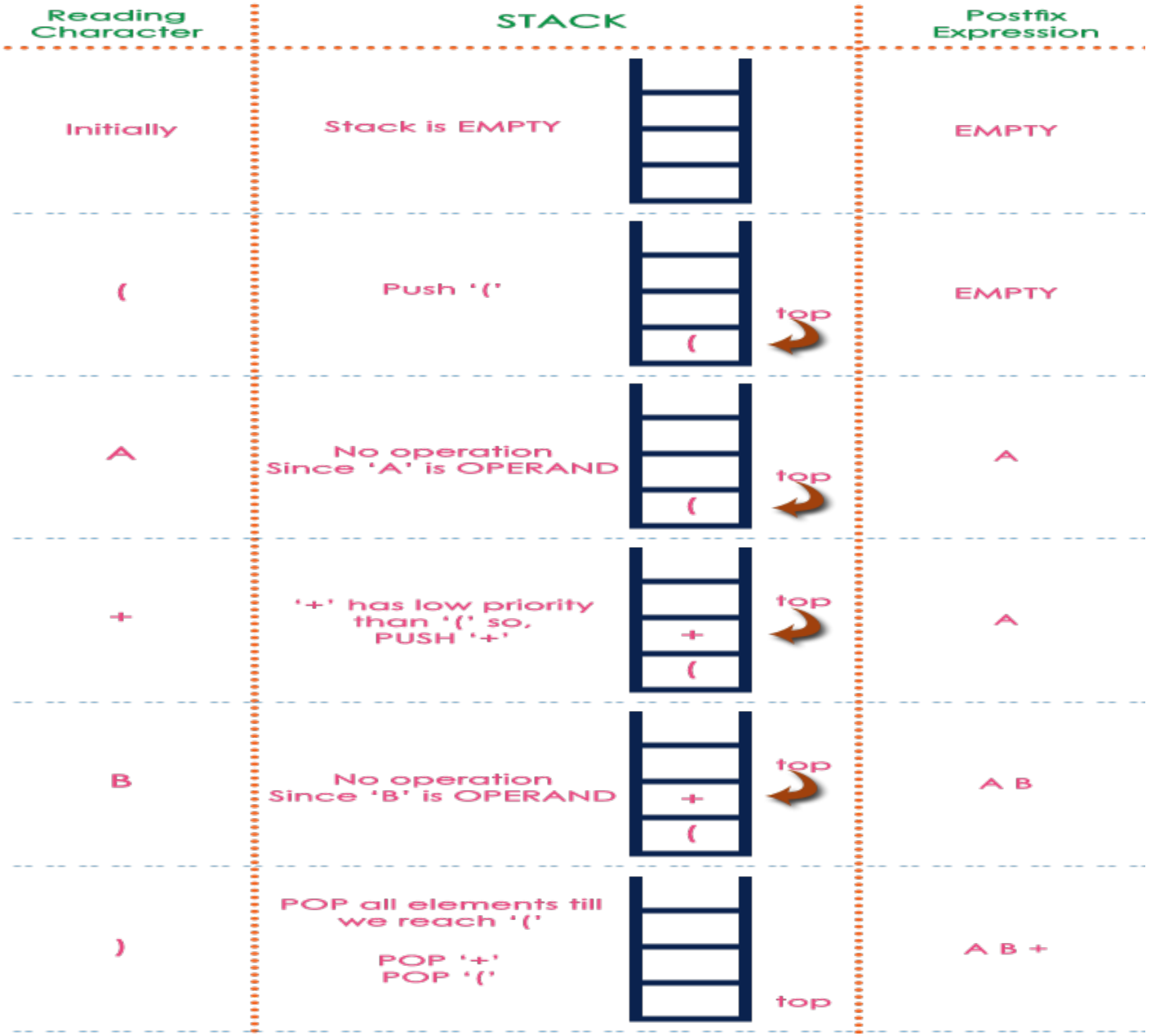
Stack	Input	Output
Empty	A+(B*C-(D/E-F)*G)*H	-
Empty	+(B*C-(D/E-F)*G)*H	A
+	(B*C-(D/E-F)*G)*H	A
+(	B*C-(D/E-F)*G)*H	A
+(	*C-(D/E-F)*G)*H	AB
+(*	C-(D/E-F)*G)*H	AB
+(*	-(D/E-F)*G)*H	ABC
+(-	(D/E-F)*G)*H	ABC*
+(-(	D/E-F)*G)*H	ABC*
+(-(	/E-F)*G)*H	ABC*D
+(-(/	E-F)*G)*H	ABC*D
+(-(/	-F)*G)*H	ABC*DE
+(-(-	F)*G)*H	ABC*DE/
+(-(-	F)*G)*H	ABC*DE/
+(-(-	)*G)*H	ABC*DE/F
+(-	*G)*H	ABC*DE/F-
+(-*	G)*H	ABC*DE/F-
+(-*	)*H	ABC*DE/F-G
+	*H	ABC*DE/F-G*-
+*	H	ABC*DE/F-G*-
+*	End	ABC*DE/F-G*-H
Empty	End	ABC*DE/F-G*-H*+

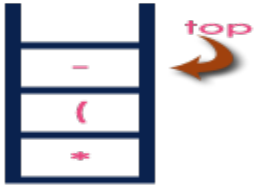
Postfix expression

ABC*DE/F-G*-H*+

Example : (A + B) * (C - D)

The given infix expression can be converted into postfix expression using Stack data Structure as follows...



*	Stack is EMPTY & '*' is Operator PUSH '*'		A B +
(	PUSH '('		A B +
C	No operation Since 'C' is OPERAND		A B + C
-	'-' has low priority than '(' so, PUSH '-'		A B + C
D	No operation Since 'D' is OPERAND		A B + C D
)	POP all elements till we reach '(' POP '-' POP '('		A B + C D -
$	POP all elements till Stack becomes Empty		A B + C D - *

2. Infix to Prefix Conversion

Following table shows the evaluation of Infix to Prefix:

Conversion from infix to prefix:

The precedence rules for converting an expression from infix to prefix are identical. The only change from postfix conversion is that traverse the expression from right to left and the operator is placed before the operands rather than after them. The prefix form of a complex expression is not the mirror image of the postfix form.

Example 1:

Convert the infix expression A + B – C into prefix expression

SYMBOL	PREFIX STRING	STACK	REMARKS
C	C		
-	C	-	
B	B C	-	
+	B C	- +	
A	A B C	- +	
End of string	- + A B C	The input is now empty. Pop the output symbols from the stack until it is empty.	

Example convert Infix expression to prefix expression = **(A+B^C)*D+E^5**

Step 1. Reverse the infix expression.

5^E+D*)C^B+A(

Step 2. Make Every '(' as ')' and every ')' as '('

5^E+D*(C^B+A)

Step 3. Convert expression to postfix form.

A+(B*C-(D/E-F)*G)*H

Step 4. Reverse the expression.

+*+A^BCD^E5

Expression	Stack	Output	Comment
5^E+D*(C^B+A)	Empty	-	Initial
^E+D*(C^B+A)	Empty	5	Print
E+D*(C^B+A)	^	5	Push
+D*(C^B+A)	^	5E	Push
D*(C^B+A)	+	5E^	Pop And Push
*(C^B+A)	+	5E^D	Print
(C^B+A)	+*	5E^D	Push
C^B+A)	+*(	5E^D	Push
^B+A)	+*(	5E^DC	Print
B+A)	+*(^	5E^DC	Push
+A)	+*(^	5E^DCB	Print
A)	+*(+	5E^DCB^	Pop And Push
)	+*(+	5E^DCB^A	Print
End	+*	5E^DCB^A+	Pop Until '('
End	Empty	5E^DCB^A+*+	Pop Every element

Result

+*+A^BCD^E5

Out put_stack = **GF+ED+C*BA-+-**

Reversing the output_stack we get prefix expression: **-+-AB*C+DE+FG**

Task b:

Evaluating the prefix expression:
Assigning the values to variables

A=7	B=6	C=5	D=4	E=3	F=2	G=1

Reading expression from right to left:

reading	operation	Stack_output
1	1	Empty
2	12	
+	12+	3
3	3 3	3
4	334	3
+	3 3 4+	3 7
5	3 7 5	3 7
*	3 7 5*	3 7 35
6	3 35 6	3 35
7	3 35 6 7	3 35
-	3 35 6 7-	3 35 1
+	3 35 1+	3 36
-	3 36 -	33

Pop stack_output : 33

Result=33

Example : 4,5,4,2,^,+,*,2,2,^,7,3,/,*,-

Step	Symbol	Operator in Stack
1	4	4
2	5	4,5
3	4	4.5.4
4	2	4,5,4,2
5	^	4,5,16
6	+	4,21
7	*	84
8	2	84,2
9	2	84,2,2
10	^	84,4
11	9	84,4,9
12	3	84,4,9,3
13	/	84,4,3
14	*	84,12
15	-	72

Que 1. Solve the following expression
5,6,2+,*,12,4,/,-

Symbol Scanned	STACK		
(1) 5	5		
(2) 6	5,	6	
(3) 2	5,	6,	2
(4) +	5,	8	
(5) *	40		
(6) 12	40,	12	
(7) 4	40,	12,	4
(8) /	40,	3	
(9) -	37		

Answer = 37

Que 2. Solve the following expression
15,3,2+,5,/,3,7,+,10,2,*

input	Stack	Output
15	15	
3	15 3	
2	15 3 2	
+	15	3+2=5
5	15 5	
/		15/5=3
3	3	
7	3 7	
+		3+7=10
10	10	
2	10 2	
*		10*2=20

Answer = 20

3. Postfix to Infix

Example: Convert Postfix to Infix efg-+he-sh-o+/*

efg-+he-sh-o+/*	NULL
fg-+he-sh-o+/*	“e”
g-+he-sh-o+/*	“f” “e”
-+he-sh-o+/*	“g” “f” “e”
+he-sh-o+/*	“f”-“g” “e”
he-sh-o+/*	“e+f-g”
e-sh-o+/*	“h” “e+f-g”
-sh-o+/*	“e” “h” “e+f-g”
sh-o+/*	“h-e” “e+f-g”
h-o+/*	“s” “h-e” “e+f-g”
-o+/*	“h” “s” “h-e” “e+f-g”
o+/*	“h-s” “h-e” “e+f-g”
+/*	“o” “s-h” “h-e” “e+f-g”
/*	“s-h+o” “h-e” “e+f-g”
*	“(h-e)/(s-h+o)” “e+f-g”
NULL	**“(e+f-g)* (h-e)/(s-h+o)”**

So, the Infix Expression is
(e+f-g)* (h-e)/(s-h+o)

4. Prefix to Infix

Example: Convert Prefix to Infix /-bc+-pqr

Expression	Stack
-bc+-pqr	Empty
/-bc+-pq	"q" "r"
/-bc+-	"p" "q" "r"
/-bc+	"p-q" "r"
/-bc	"p-q+r"
/-b	"c" "p-q+r"
/-	"b" "c" "p-q+r"
/	"b-c" "p-q+r"
NULL	**"((b-c)/((p-q)+r))"**

So, the Infix Expression is
((b-c)/((p-q)+r))

Unit 5. Queues

5.1 Introduction

5.2 Representation - Static & Dynamic

5.3 Primitive Operation on Queues

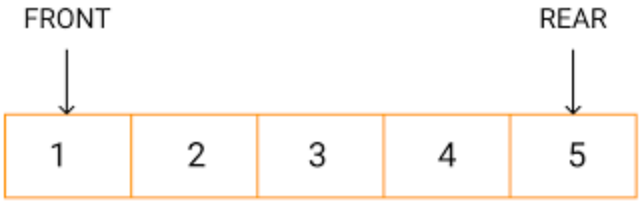
5.4 Circular Queue ,Priority Queue

5.5 Concept of doubly ended queue (dequeue)

5.1 Introduction

What is Queue?

- Queue is a linear data structure where the first element is inserted from one end called **REAR** and deleted from the other end called as **FRONT**.
- **Front** points to the **beginning** of the queue and **Rear** points to the **end** of the queue.
- Queue follows the **FIFO (First - In - First Out)** structure.
- According to its FIFO structure, element inserted first will also be removed first.
- In a queue, one end is always used to insert data and the other is used to delete data.



5.2 Representation of Queue

- **Array** is the easiest way to implement a queue. Queue can be also implemented using Linked List or Stack.

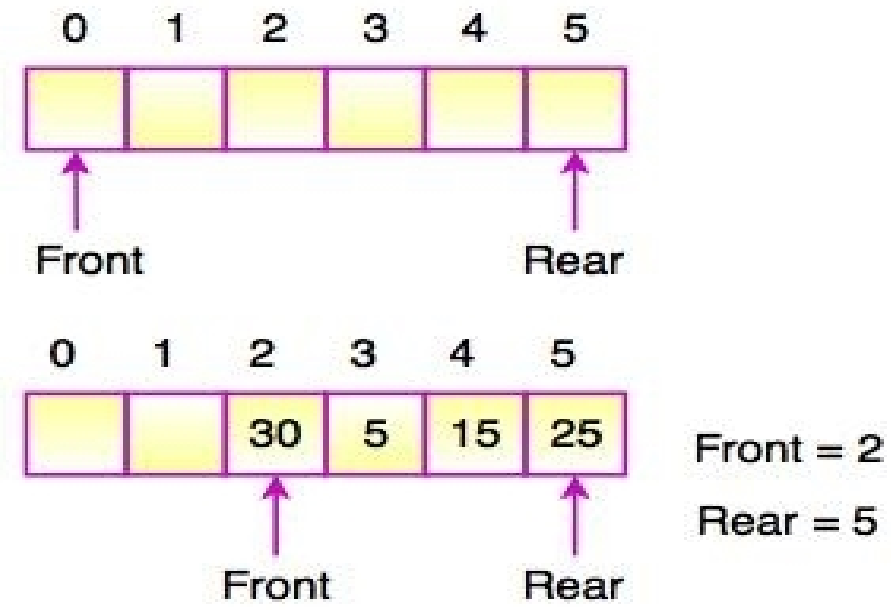
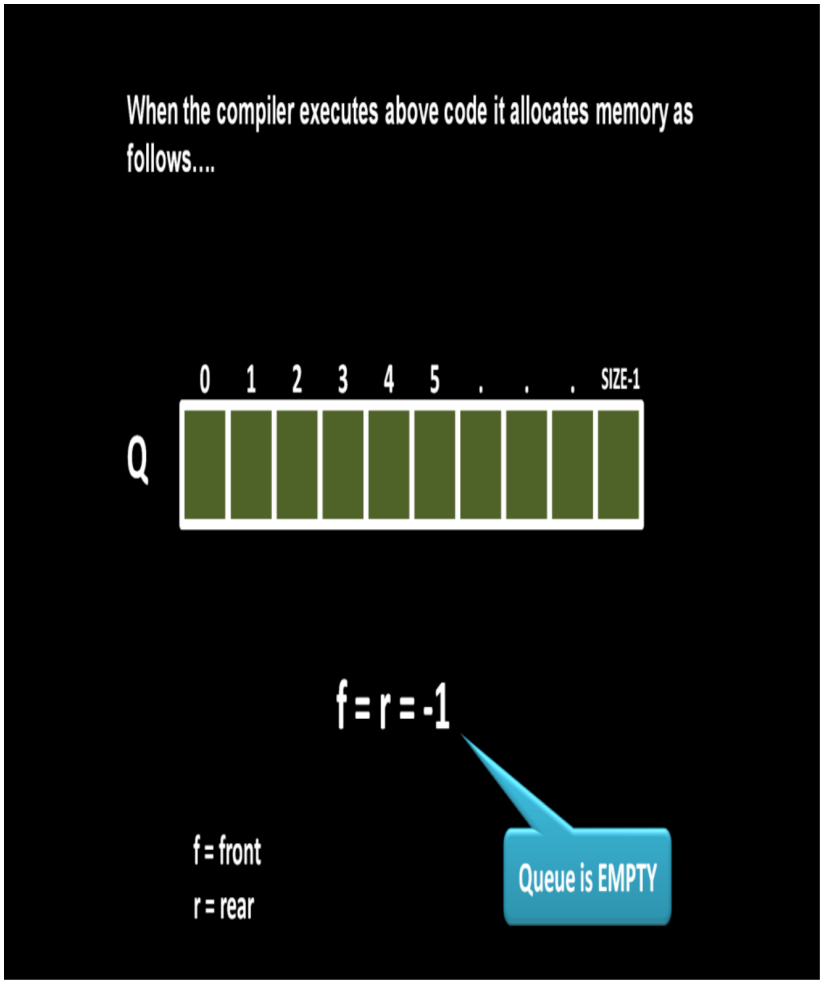


Fig. Implementation of Queue using Array

In the above diagram, Front and Rear of the queue point at the first index of the array. (Array index starts from 0).

While adding an element into the queue, the Rear keeps on moving ahead and always points to the position where the next element will be inserted. Front remains at the first index.

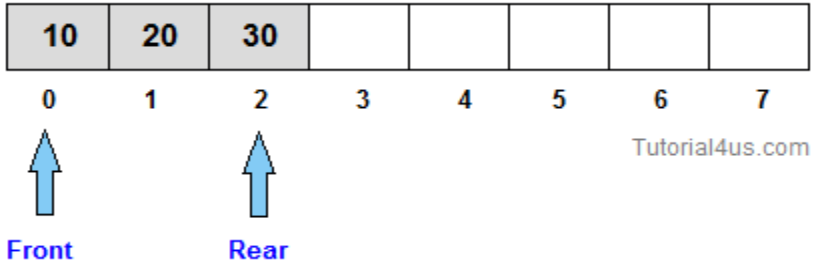


5.3 Primitive operation on a queue

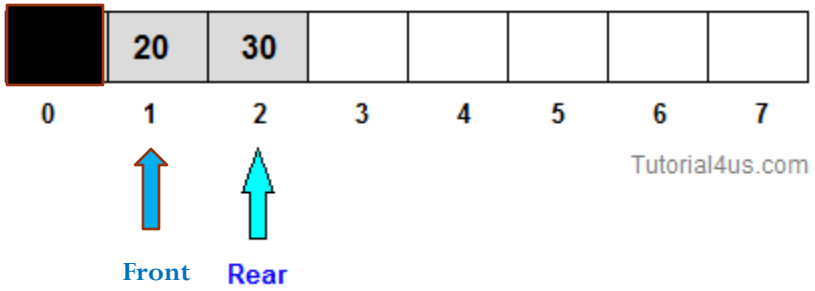
The basic operation that can be perform on queue are;

1. Insert an Element in a Queue.
2. Delete an Element from the Queue.

1. Insert an element in a queue.



2. Delete an Element from the Queue.



Queue Applications

- **Real-World Applications**
  - Buy a movie ticket
  - Check out at a bookstore
  - Bank / ATM
  - Call an airline
  - Cashier lines in any store
- **Computer Science Applications**
  - OS task scheduling
  - Print lines of a document
  - Printer shared between computers
  - Convert digit strings to decimal
  - Shared resource usage (CPU, memory access, **...**)

Functions of Queue

insert()	delete()	display()

```
insert()
{
   int add_item;
   if (rear == MAX - 1)
    {
 printf("Queue Overflow \n");
   }
 else
   {
      if (front == - 1)
      {
      front = 0;
     }
  printf("Inset the element in queue : ");
      scanf("%d", &add_item);
      rear = rear + 1;
     queue_array[rear] = add_item;
   }
}
```

```
delete()
{
 if (front == - 1 || front > rear)
    {
printf("Queue Underflow \n");
          }
    else
    {
 printf("Deleted Element is :
%d\n", queue_array[front]);
       front = front + 1;
    }
}
```

```
display()
{
    int i;
    if (front == - 1)
    {
    printf("Queue is empty \n");
     }
 else
    {
   for (i = front; i <= rear;
i++)
    {
     printf("%d \n", queue_array[i]);
    }
    }
}
```

Program of Queue using Array

```
#include <stdio.h>
#define MAX 50
int queue_array[MAX];
int rear = - 1;
int front = - 1;
main()
{
   int choice;
   while (1)
   {
      printf("1.Insert \n");
      printf("2.Delete\n");
      printf("3.Display \n");
      printf("4.Exit \n");
      printf("Enter your choice : ");
      scanf("%d", &choice);
      switch (choice)
      {
         case 1:
         insert();
         break;
         case 2:
         delete();
         break;
         case 3:
         display();
         break;
         case 4:
         exit(1);
         default:
         printf("Inavlid choice \n");
      }
   }
}
```

```
insert()
{
   int add_item;
   if (rear == MAX - 1)
   {
   printf("Queue Overflow \n");
   }
 else
   {
      if (front == - 1)
         {
            front = 0;
         }
      printf("Inset the element in queue : ");
      scanf("%d", &add_item);
      rear = rear + 1;
      queue_array[rear] = add_item;
   }
}
delete()
{
   if (front == - 1 || front > rear)
   {
      printf("Queue Underflow \n");
    }
   else
   {
      printf("Deleted Element is : %d\n",
queue_array[front]);
      front = front + 1;
   }
}
```

```
display()
{
   int i;
   if (front == - 1)
    {
      printf("Queue is empty \n");
    }
   else
   {
       for (i = front; i <= rear; i++)
            {
              printf("%d ", queue_array[i]);
    }}}
```

1. Insert Element in Queue

```
1.Insert
2.Delete
3.Display
4.Exit
Enter your choice : 1
Inset the element in queue : 10
1.Insert
2.Delete
3.Display
4.Exit
Enter your choice : 1
Inset the element in queue : 20
```

2. Display Element

```
1.Insert
2.Delete
3.Display
4.Exit
Enter your choice : 3
Queue is :
10 20 30 40
```

3. Delete Element

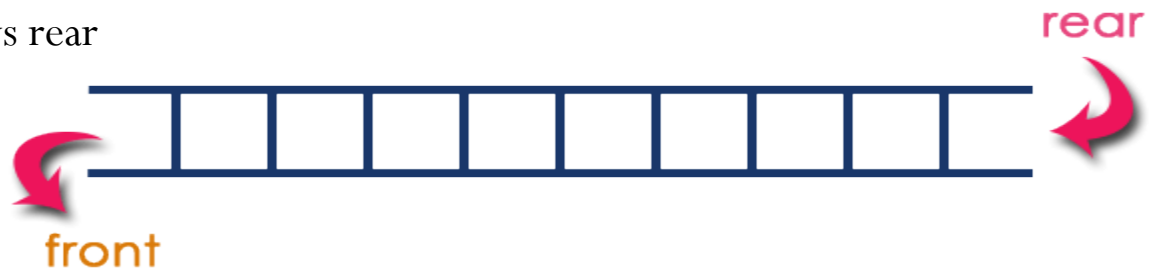
```
1.Insert
2.Delete
3.Display
4.Exit
Enter your choice : 2
Deleted Element is : 10
```

Types of Queue in Data Structure

- **There are four types of Queue:**
  1. Linear Queue
  2. Circular Queue
  3. Priority Queue
  4. Dequeue (Double Ended Queue)1. Simple Queue

1. Linear Queue

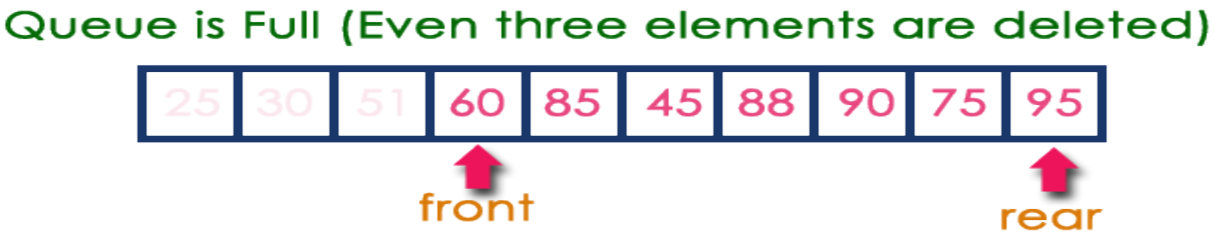
In this queue the elements are arranged into sequential manner. Such that front position is always less than or equal to the rear position. When an element is added, rear is incremented and when an element is removed front is advanced. Thus front always rear



2. Circular Queue

- In this queue , the elements are arranged in a sequential manner but can logically be regarded as circularly arranged.
- In a normal Queue Data Structure, we can insert elements until queue becomes full. But once the queue becomes full, we can not insert the next element until all the elements are deleted from the queue.
- For example, consider the queue below…The queue after inserting all the elements into it is as follows…

Now consider the following situation after deleting three elements from the queue…



This situation also says that Queue is Full and we cannot insert the new element because '**rear**' is still at last position. In the above situation, even though we have empty positions in the queue we can not make use of them to insert the new element. This is the major problem in a normal queue data structure. To overcome this problem we use a circular queue data structure.

- In a circular queue, all nodes are treated as circular. Last node is connected back to the first node.
- Circular queue is also called as **Ring Buffer.**
- It is an abstract data type.
- Circular queue contains a collection of data which allows insertion of data at the end of the queue and deletion of data at the beginning of the queue.

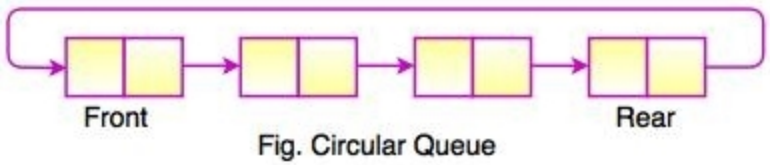


Fig. Circular Queue

The above figure shows the structure of circular queue. It stores an element in a circular way and performs the operations according to its FIFO structure.

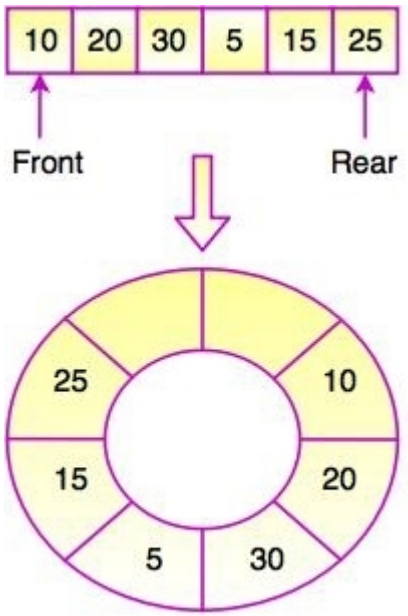


Fig. Circular Queue

PRIORITY QUEUE

1. It is collection of elements where elements are stored according to the their priority levels.
2. Inserting and removing of elements from queue is decided by the priority of the elements.
3. An element of the higher priority is processed first.
4. Two element of same priority are processed on first-come-first-served basis.

Operation	Priority Queue	Return value
Insert(G)	G	
Insert(O)	G O	
Insert(M)	G O M	
deleteHighestPriority()	G M	O
Insert(A)	G M A	
deleteHighestPriority()	G A	M

Applications of Priority Queue:

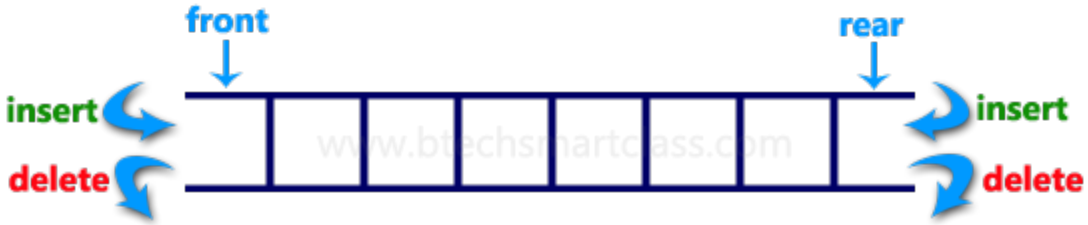
1) CPU Scheduling

2) Graph algorithms like Dijkstra's shortest path algorithm, Prim's Minimum Spanning Tree, etc

3) All queue applications where priority is involved.

5.5 Concept of Dequeue (Double Ended Queue)

Double Ended Queue is also a Queue data structure in which the insertion and deletion operations are performed at both the ends (**front** and **rear**). That means, we can insert at both front and rear positions and can delete from both front and rear positions.

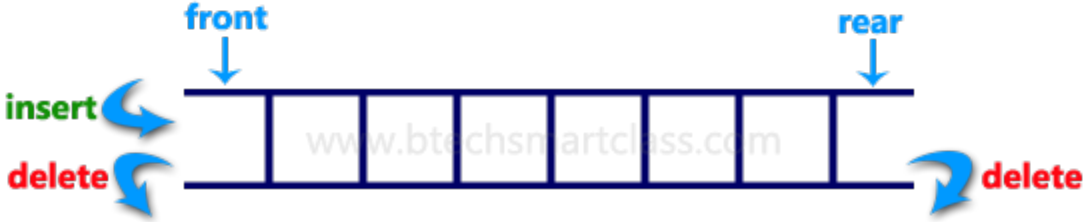


Double Ended Queue can be represented in TWO ways, those are as follows…

1. Input Restricted Double Ended Queue
2. Output Restricted Double Ended Queue

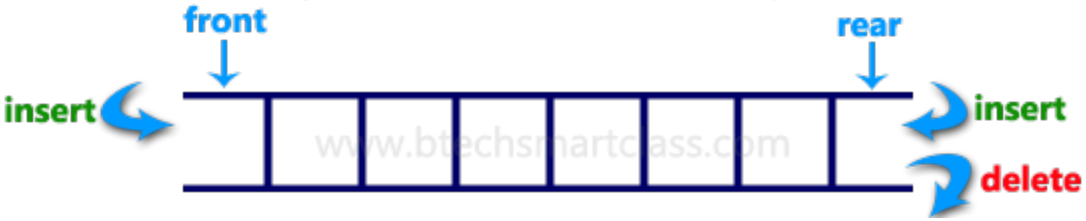
1. Input Restricted Double Ended Queue

In input restricted double-ended queue, the insertion operation is performed at only one end and deletion operation is performed at both the ends.



2. Output Restricted Double Ended Queue

In output restricted double ended queue, the deletion operation is performed at only one end and insertion operation is performed at both the ends.



6. Trees

6.1Concept and Terminology

6.2 Binary Tree .Binary search tree

6.3 Representation – static and dynamic

6.4 Operations on Binary Tree create ,Inset, delete, counting leaf and non leaf nodes, total nodes

6.5 Tree Traversal (Pre-order In-order ,Post-order)

6.6 Application- Heap sort)

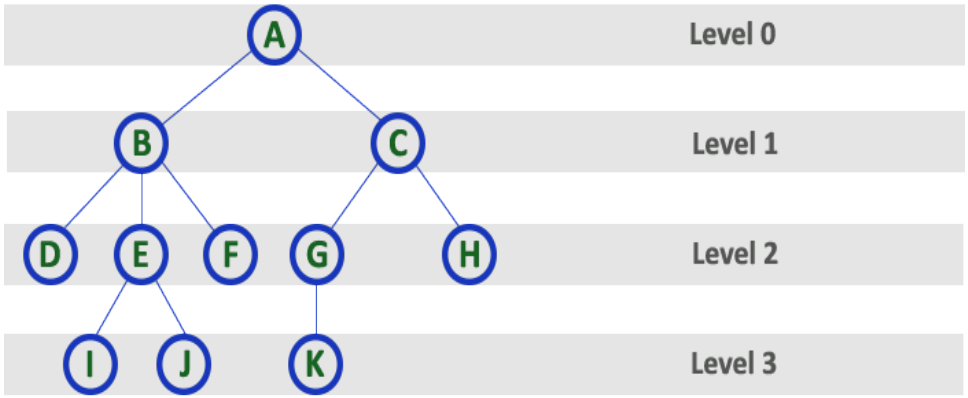
6.7 Height Balanced Tree-AVL Tree-Rotations -Example

6.1 Concept and Terminology

What is Tree?

- Tree is a non linear data structure which represents hierarchical relationship among its elements.
- Along with information storage tree as a data structure is efficient with respect to operations such as Insertion, Deletion, Searching as compared to linear data structure.
- **Properties of Tree :**

1. There exists unique path between every two vertices.
2. The number of vertices is one more than the no of edges in tree
3. A tree with two or more vertices has at least two leaves.



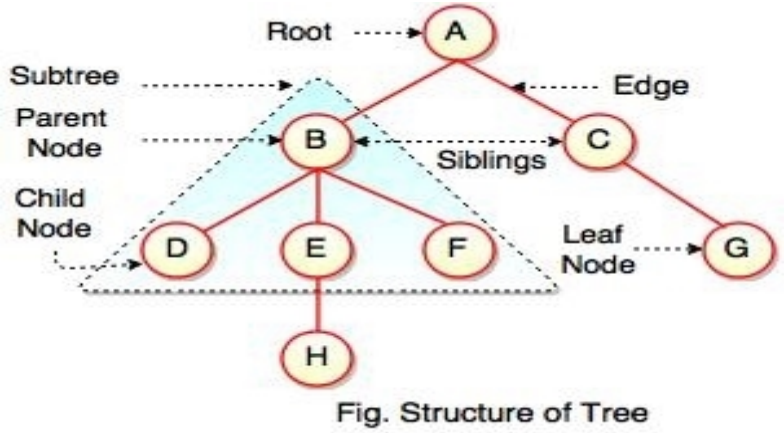
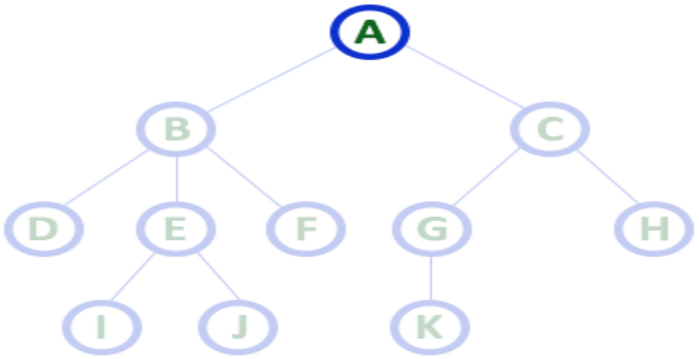


Fig. Structure of Tree

Tree - Terminology

1. Root

In a tree data structure, the first node is called as **Root Node**. Every tree must have a root node. We can say that the root node is the origin of the tree data structure. In any tree, there must be only one root node. We never have multiple root nodes in a tree.

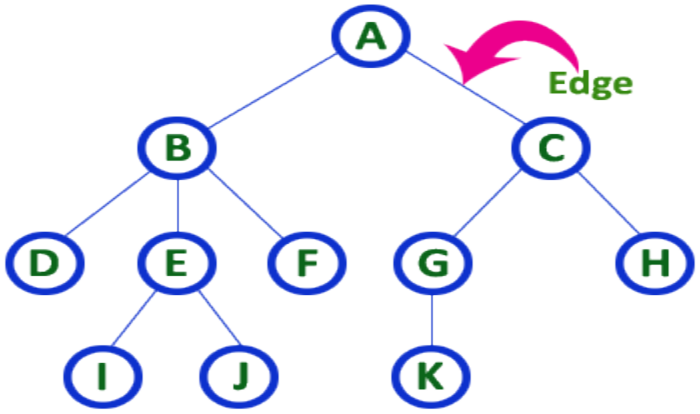


Here 'A' is the 'root' node

- In any tree the first node is called as ROOT node

2. Edge

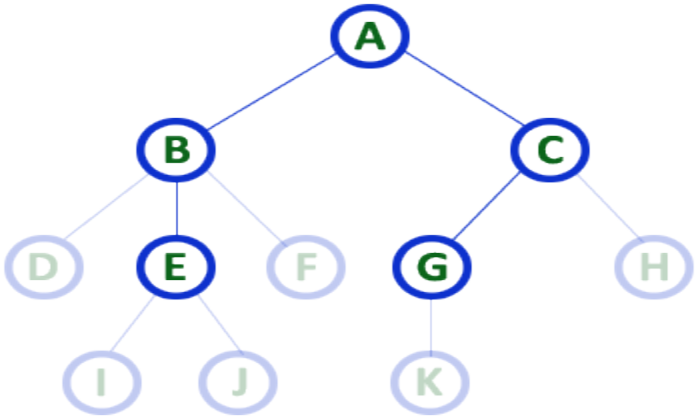
In a tree data structure, the connecting link between any two nodes is called as **EDGE**. In a tree with '**N**' number of nodes there will be a maximum of '**N-1**' number of edges.



- In any tree, 'Edge' is a connecting link between two nodes.

3. Parent

In a tree data structure, the node which is a predecessor of any node is called as **PARENT NODE**. In simple words, the node which has a branch from it to any other node is called a parent node. Parent node can also be defined as "**The node which has child / children**".

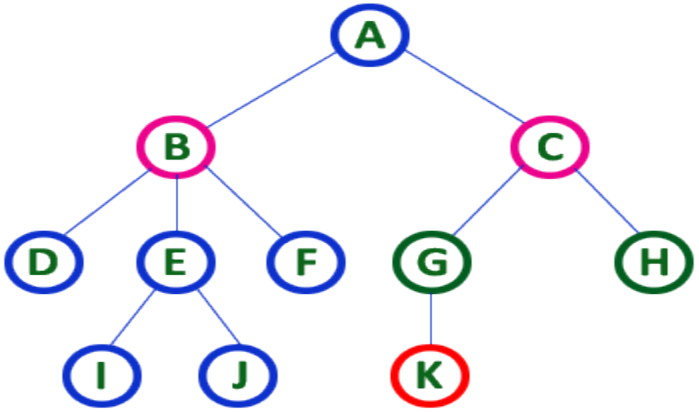


Here A, B, C, E & G are Parent nodes

- In any tree the node which has child / children is called 'Parent'
- A node which is predecessor of any other node is called 'Parent'

4. Child

In a tree data structure, the node which is descendant of any node is called as **CHILD Node**. In simple words, the node which has a link from its parent node is called as child node. In a tree, any parent node can have any number of child nodes. In a tree, all the nodes except root are child nodes.

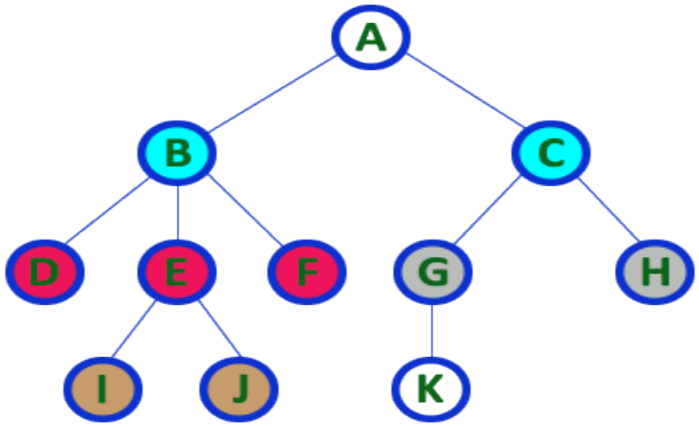


Here B & C are Children of A
Here G & H are Children of C
Here K is Child of G

- descendant of any node is called as CHILD Node

5. Siblings

In a tree data structure, nodes which belong to same Parent are called as **SIBLINGS**. In simple words, the nodes with the same parent are called Sibling nodes.



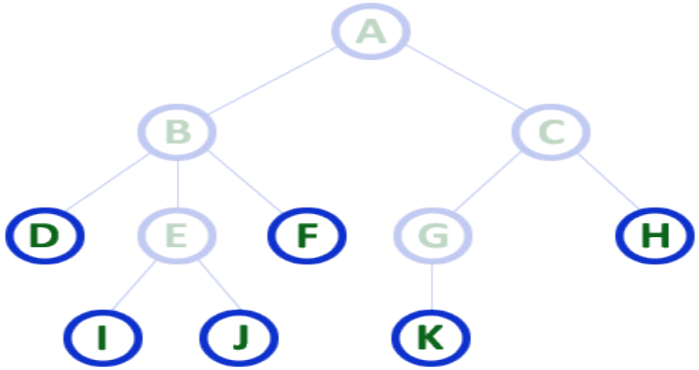
Here B & C are Siblings
Here D E & F are Siblings
Here G & H are Siblings
Here I & J are Siblings

- In any tree the nodes which has same Parent are called 'Siblings'
- The children of a Parent are called 'Siblings'

6. Leaf

In a tree data structure, the node which does not have a child is called as **LEAF Node**. In simple words, a leaf is a node with no child.

In a tree data structure, the leaf nodes are also called as **External Nodes**. External node is also a node with no child. In a tree, leaf node is also called as '**Terminal**' node.



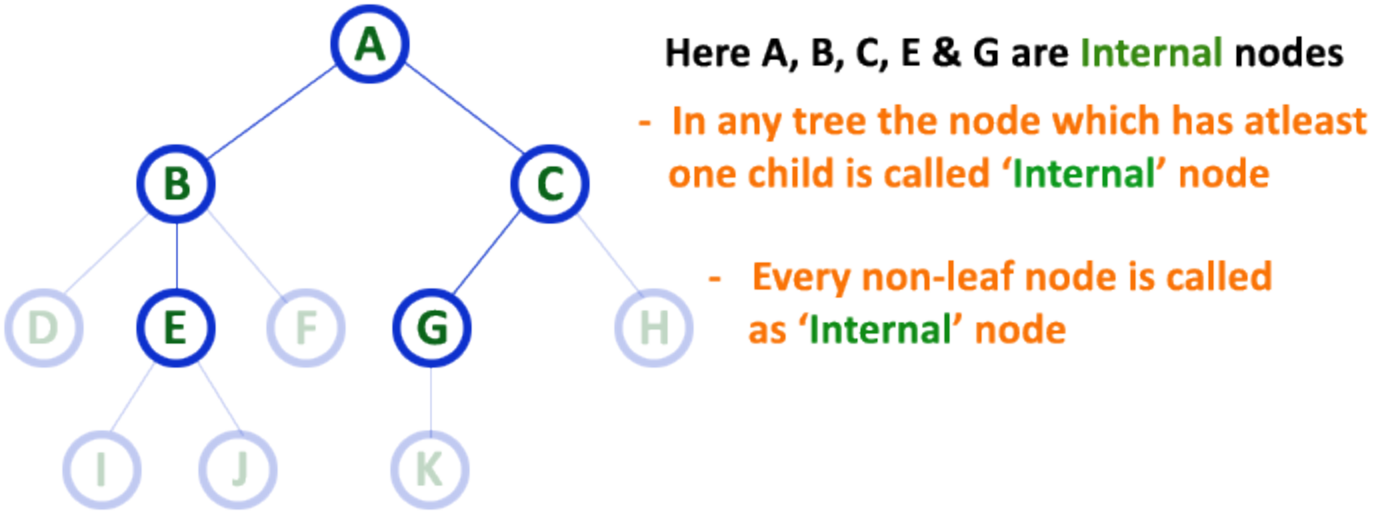
Here D, I, J, F, K & H are Leaf nodes

- In any tree the node which does not have children is called 'Leaf'
- A node without successors is called a 'leaf' node

7. Internal Nodes

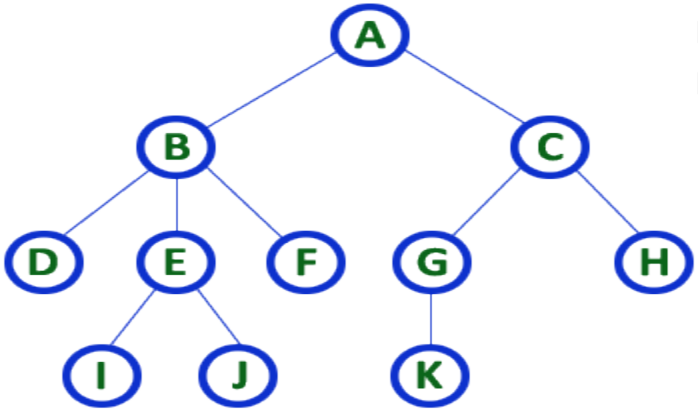
In a tree data structure, the node which has atleast one child is called as **INTERNAL Node**. In simple words, an internal node is a node with atleast one child.

In a tree data structure, nodes other than leaf nodes are called as **Internal Nodes**. **The root node is also said to be Internal Node** if the tree has more than one node. Internal nodes are also called as '**Non-Terminal**' nodes.



8. Degree

In a tree data structure, the total number of children of a node is called as **DEGREE** of that Node. In simple words, the Degree of a node is total number of children it has. The highest degree of a node among all the nodes in a tree is called as '**Degree of Tree**'



Here Degree of B is 3

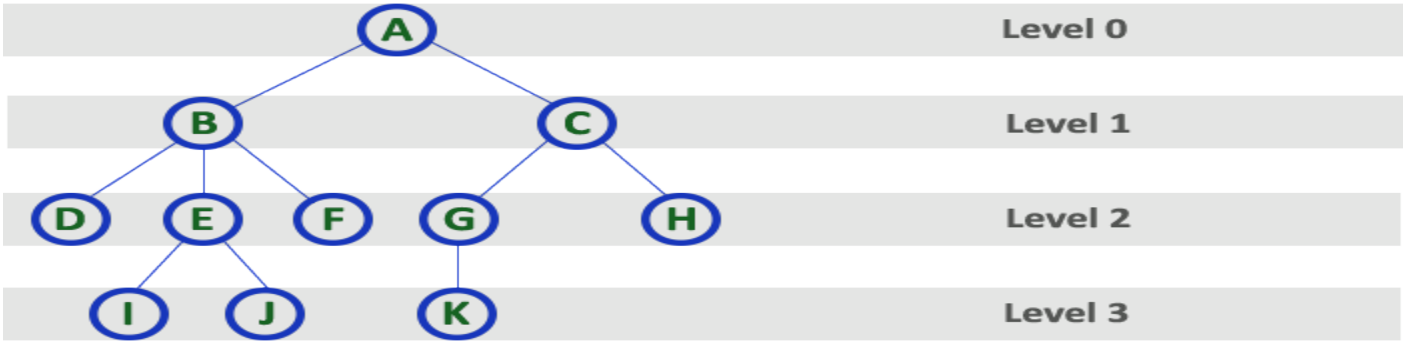
Here Degree of A is 2

Here Degree of F is 0

- In any tree, 'Degree' of a node is total number of children it has.

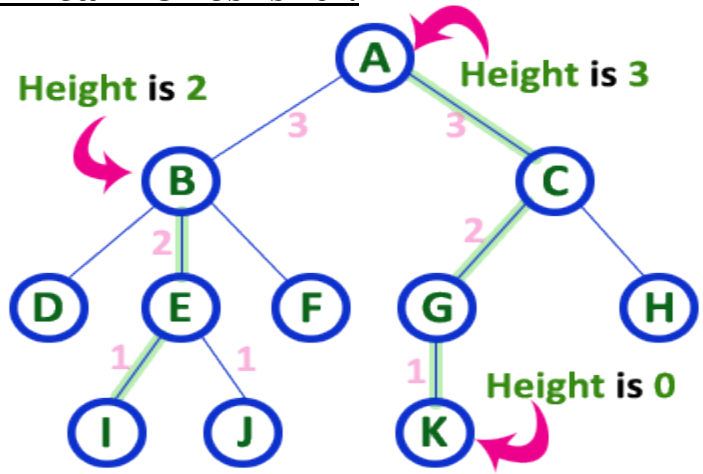
9. Level

In a tree data structure, the root node is said to be at Level 0 and the children of root node are at Level 1 and the children of the nodes which are at Level 1 will be at Level 2 and so on… In simple words, in a tree each step from top to bottom is called as a Level and the Level count starts with '0' and incremented by one at each level (Step).



10. Height

In a tree data structure, the total number of edges from leaf node to a particular node in the longest path is called as **HEIGHT** of that Node. In a tree, height of the root node is said to be **height of the tree**. In a tree, **height of all leaf nodes is '0'.**

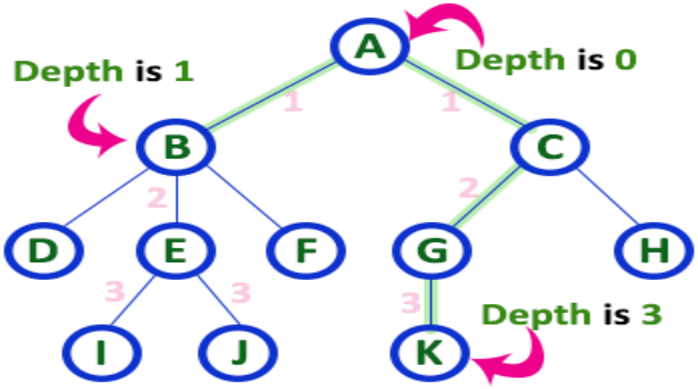


Here Height of tree is 3

- In any tree, 'Height of Node' is total number of Edges from leaf to that node in longest path.
- In any tree, 'Height of Tree' is the height of the root node.

11. Depth

In a tree data structure, the total number of egdes from root node to a particular node is called as **DEPTH** of that Node. In a tree, the total number of edges from root node to a leaf node in the longest path is said to be **Depth of the tree**. In simple words, the highest depth of any leaf node in a tree is said to be depth of that tree. In a tree, **depth of the root node is '0'.**

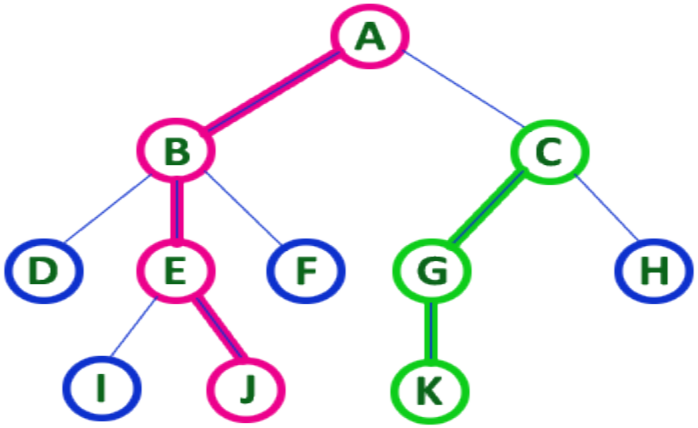


Here Depth of tree is 3

- In any tree, 'Depth of Node' is total number of Edges from root to that node.
- In any tree, 'Depth of Tree' is total number of edges from root to leaf in the longest path.

12. Path

In a tree data structure, the sequence of Nodes and Edges from one node to another node is called as **PATH** between that two Nodes. **Length of a Path** is total number of nodes in that path. In below example **the path A - B - E - J has length 4**.



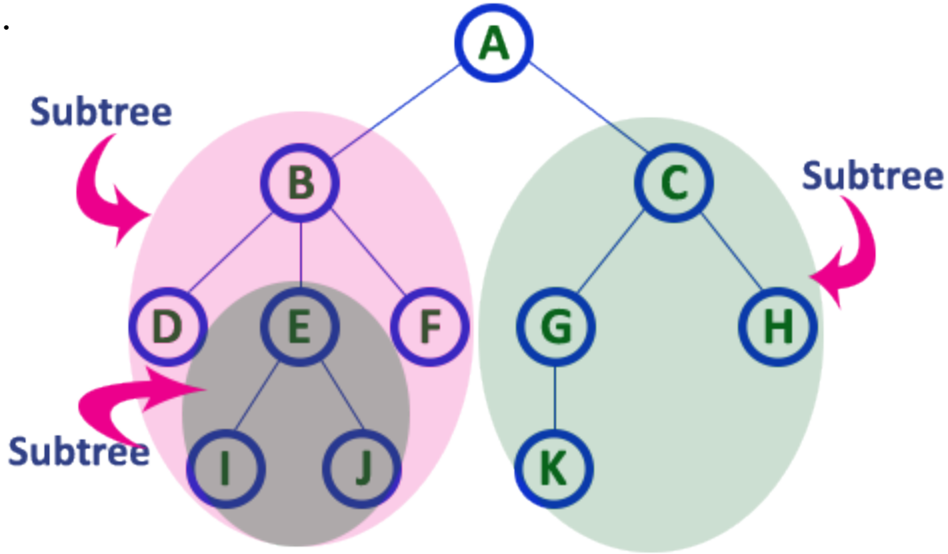
- In any tree, 'Path' is a sequence of nodes and edges between two nodes.

Here, 'Path' between A & J is
A - B - E - J

Here, 'Path' between C & K is
C - G - K

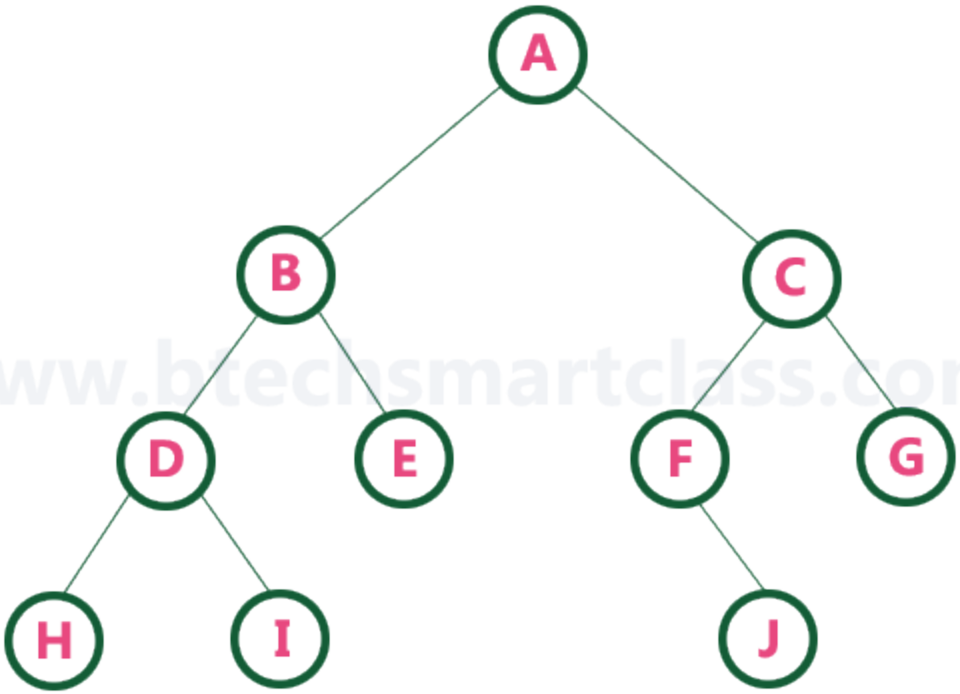
13. Sub Tree

In a tree data structure, each child from a node forms a subtree recursively. Every child node will form a subtree on its parent node.



6.2 Binary Tree and Binary Search Tree

- In a normal tree, every node can have any number of children. A binary tree is a special type of tree data structure in which every node can have a **maximum of 2 children**. One is known as a left child and the other is known as right child.
- **A tree in which every node can have a maximum of two children is called Binary Tree.**
- In a binary tree, every node can have either 0 children or 1 child or 2 children but not more than 2 children



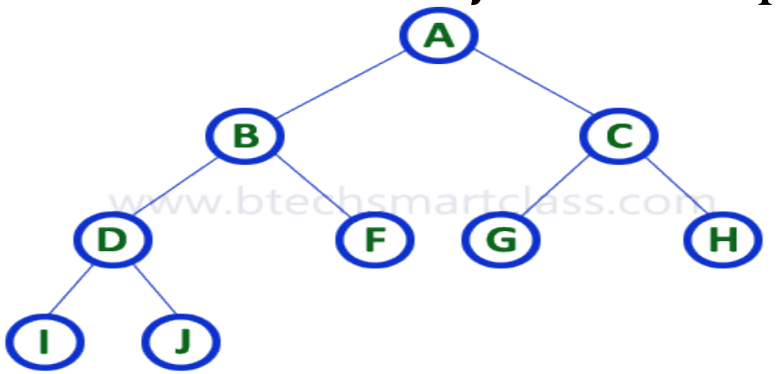
Type of Binary Tree

There are different types of binary trees and they are…

1. Strictly Binary Tree

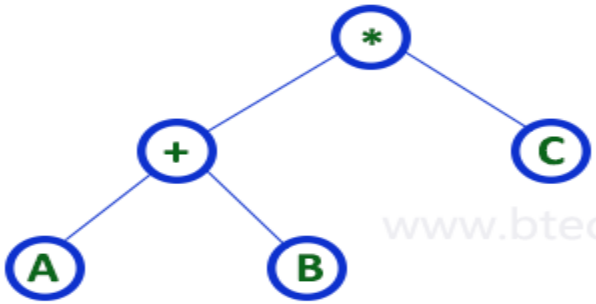
In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none. That means every internal node must have exactly two children. A strictly Binary Tree can be defined as follows…

Strictly binary tree is also called as **Full Binary Tree** or **Proper Binary Tree** or **2-Tree**

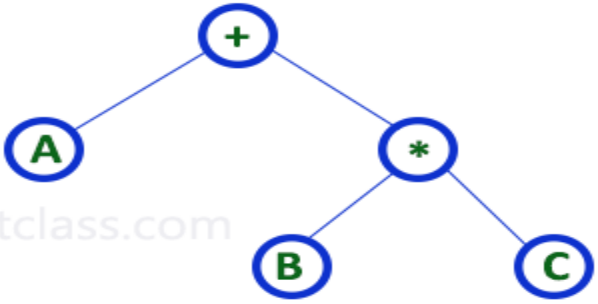


Strictly binary tree data structure is used to represent mathematical expressions.

Example



(A + B) * C



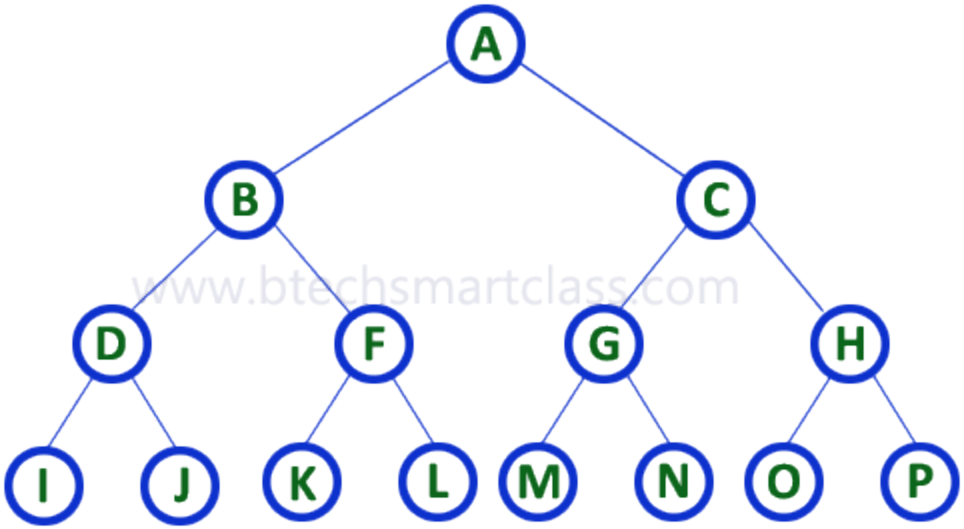
A + B * C

2. Complete Binary Tree

In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none and in complete binary tree all the nodes must have exactly two children and at every level of complete binary tree there must be 2^{level} number of nodes. For example at level 2 there must be $2^2 = 4$ nodes and at level 3 there must be $2^3 = 8$ nodes.

A binary tree in which every internal node has exactly two children and all leaf nodes are at same level is called Complete Binary Tree.

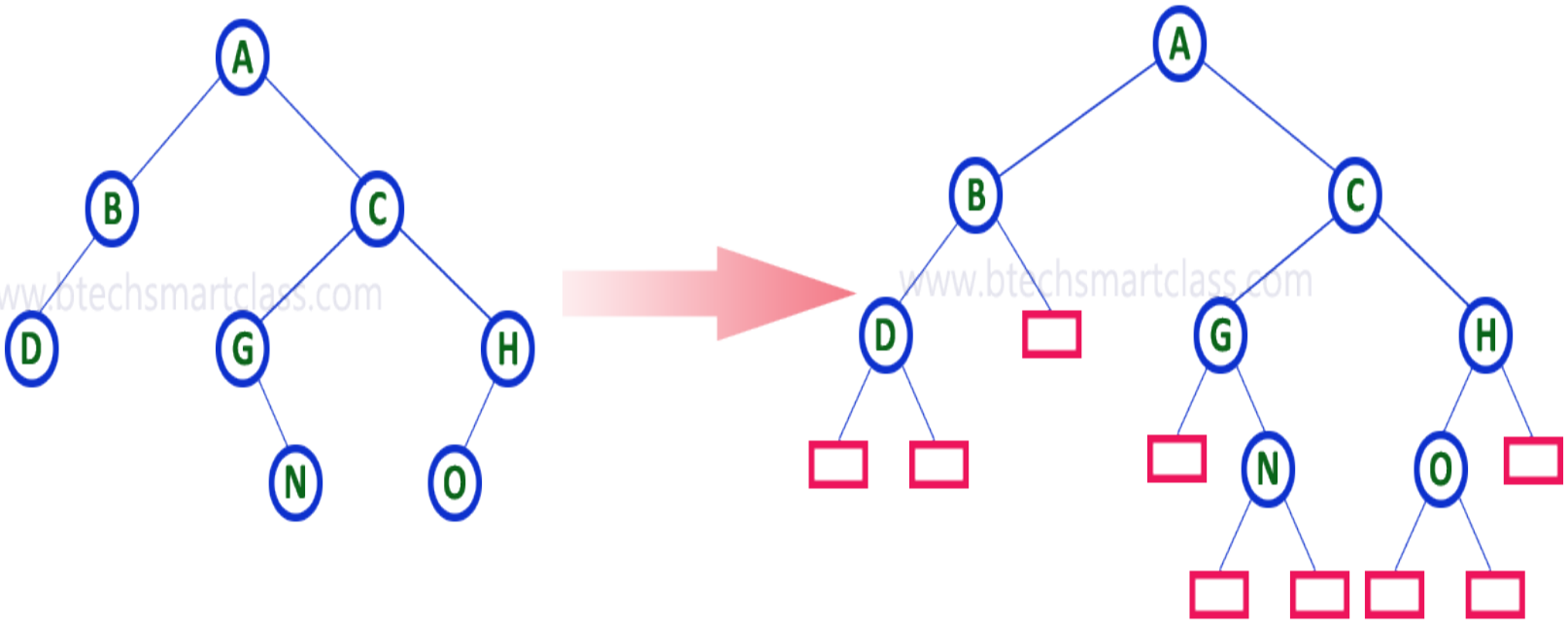
Complete binary tree is also called as **Perfect Binary Tree**



3. Extended Binary Tree

A binary tree can be converted into Full Binary tree by adding dummy nodes to existing nodes wherever required.

The full binary tree obtained by adding dummy nodes to a binary tree is called as Extended Binary Tree.



4 Skewed Binary Tree

- If a tree which is dominated by left child node or right child node, is said to be a **Skewed Binary Tree**.
- In a skewed binary tree, all nodes except one have only one child node. The remaining node has no child.

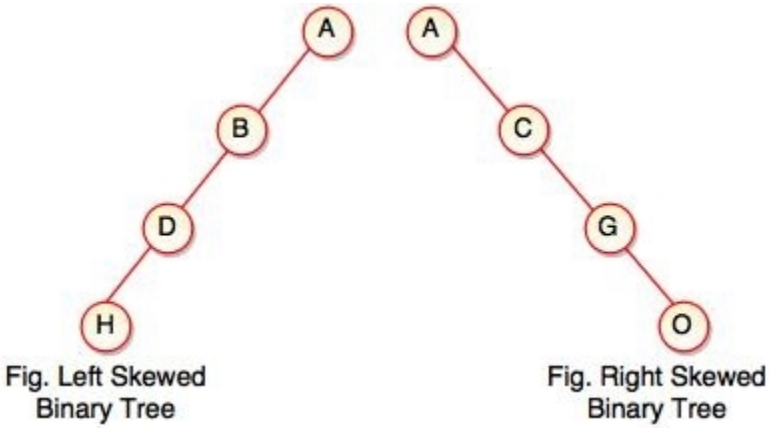


Fig. Left Skewed Binary Tree

Fig. Right Skewed Binary Tree

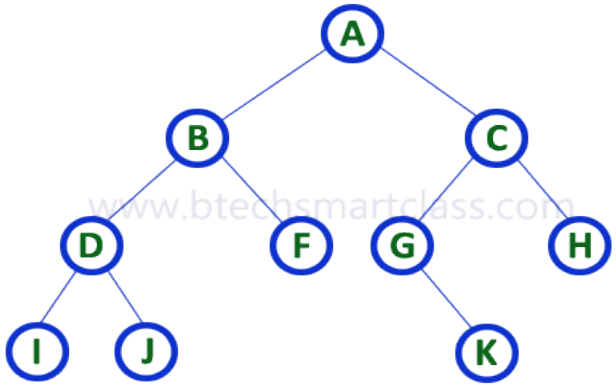
- In a left skewed tree, most of the nodes have the left child without corresponding right child.
- In a right skewed tree, most of the nodes have the right child without corresponding left child.

6.3 Binary Tree Representations

- A binary tree data structure is represented using two methods. Those methods are as follows…

1. **Array Representation**
2. **Linked List Representation**

Consider the following binary tree…



1. Array Representation of Binary Tree

In array representation of a binary tree, we use one-dimensional array (1-D Array) to represent a binary tree.

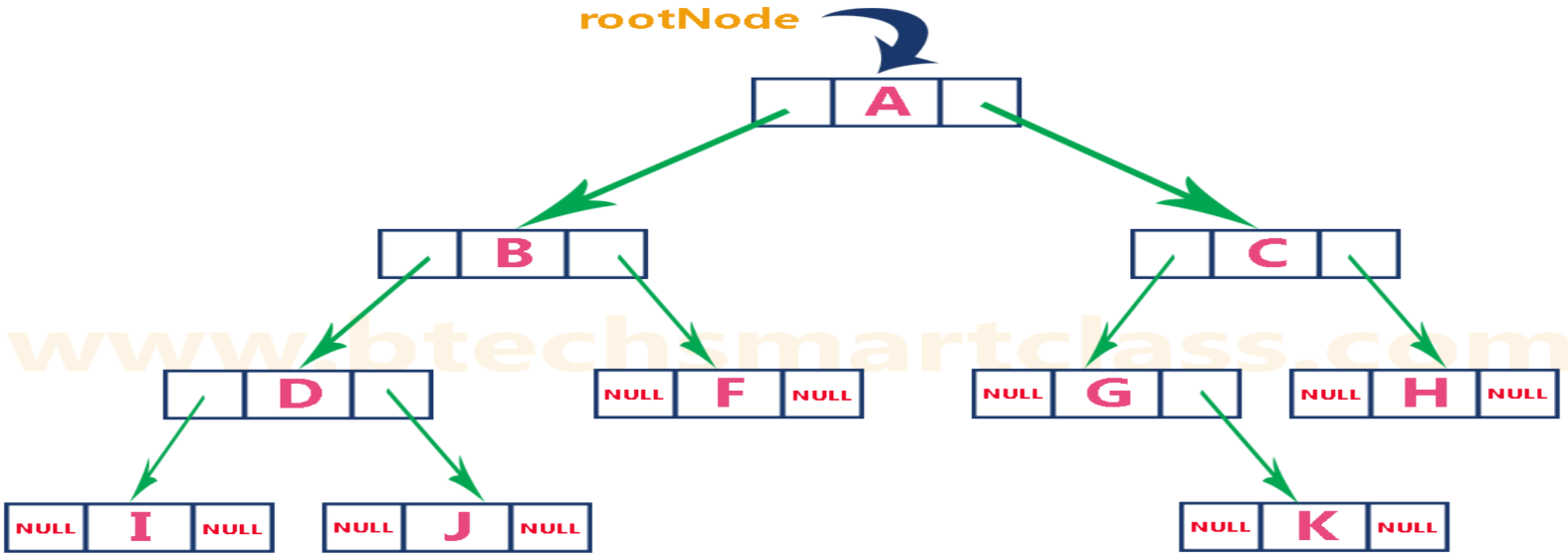
Consider the above example of a binary tree and it is represented as follows…

2. Linked List Representation of Binary Tree

We use a double linked list to represent a binary tree. In a double linked list, every node consists of three fields. First field for storing left child address, second for storing actual data and third for storing right child address.

In this linked list representation, a node has the following structure…

The above example of the binary tree represented using Linked list representation is shown as follows..



Binary Search Tree

- Binary search tree is a binary tree which has special property called BST.
- BST property is given as follows:
- **For all nodes A and B,**

  I. If B belongs to the left subtree of A, the key at B is less than the key at A.

  II. If B belongs to the right subtree of A, the key at B is greater than the key at A.

  Each node has following attributes:

  I. Parent (P), left, right which are pointers to the parent (P), left child and right child respectively.

  II. Key defines a key which is stored at the node.

Definition:

"Binary Search Tree is a binary tree where each node contains only smaller values in its left subtree and only larger values in its right subtree."

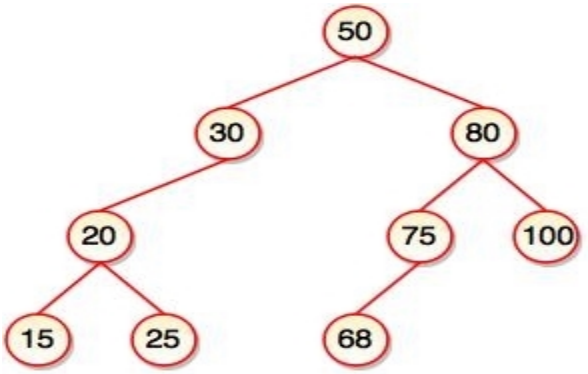
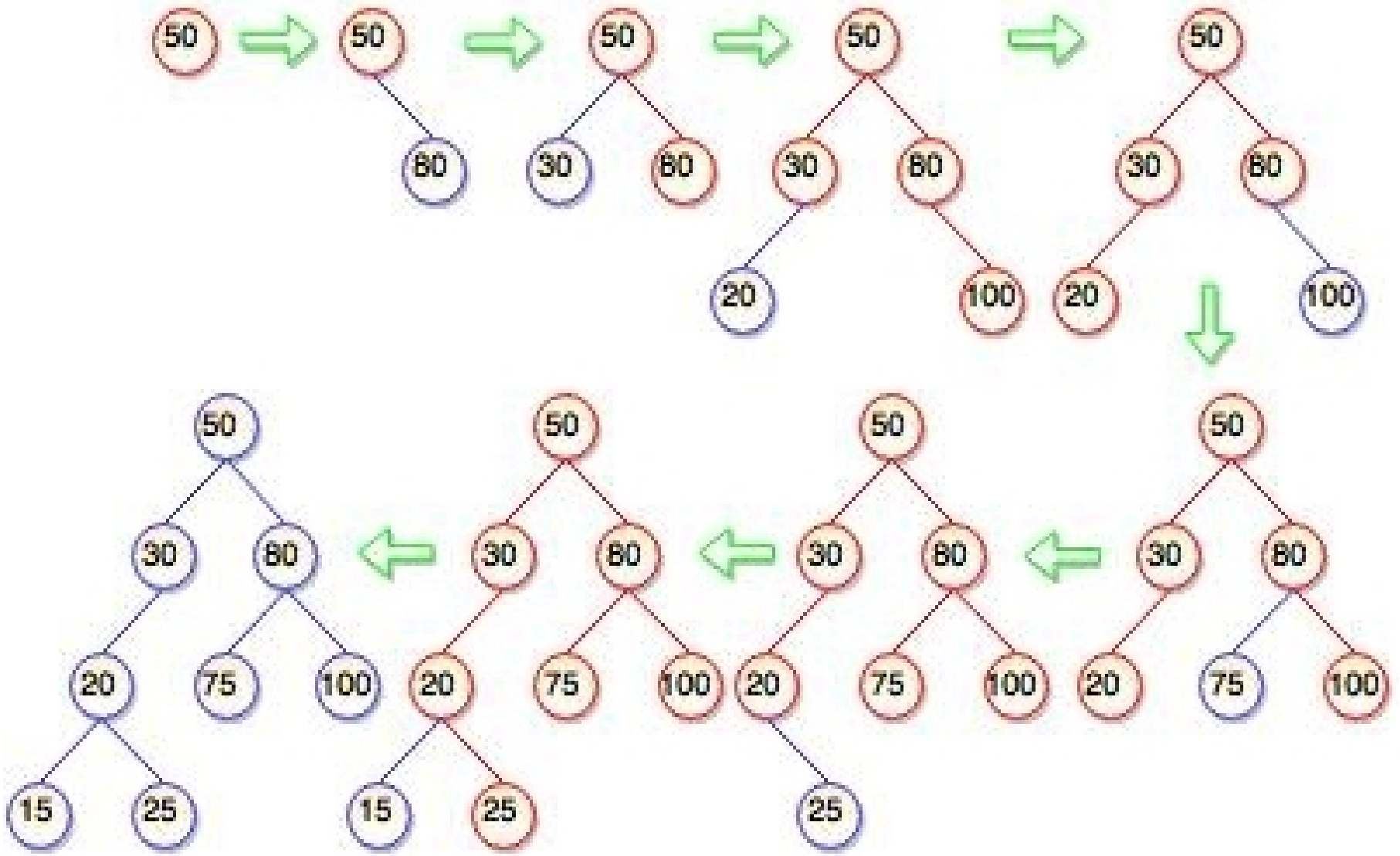


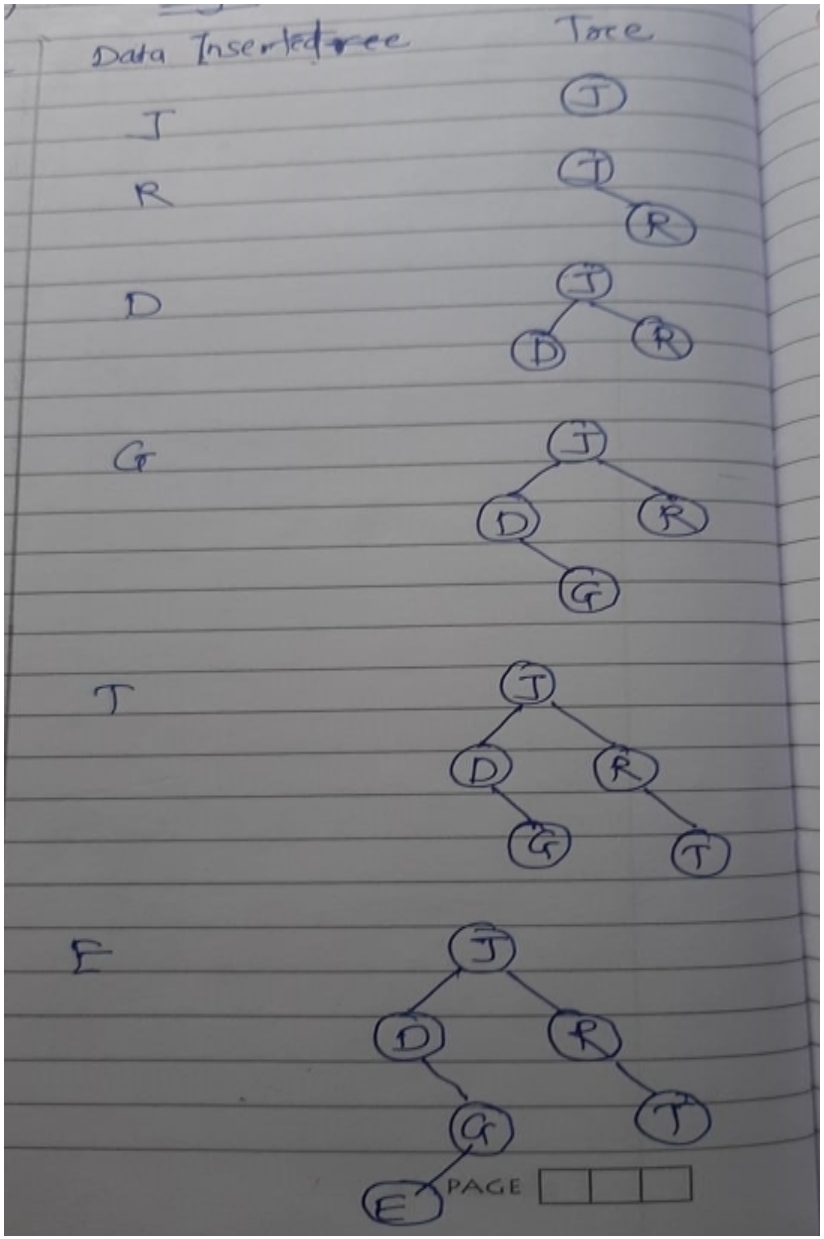
Fig. Binary Search Tree

- The tree represents binary search tree (BST) where left subtree of every node contains smaller values and right subtree of every node contains larger value.
- Binary Search Tree (BST) is used to enhance the performance of binary tree.
- It focuses on the search operation in binary tree.

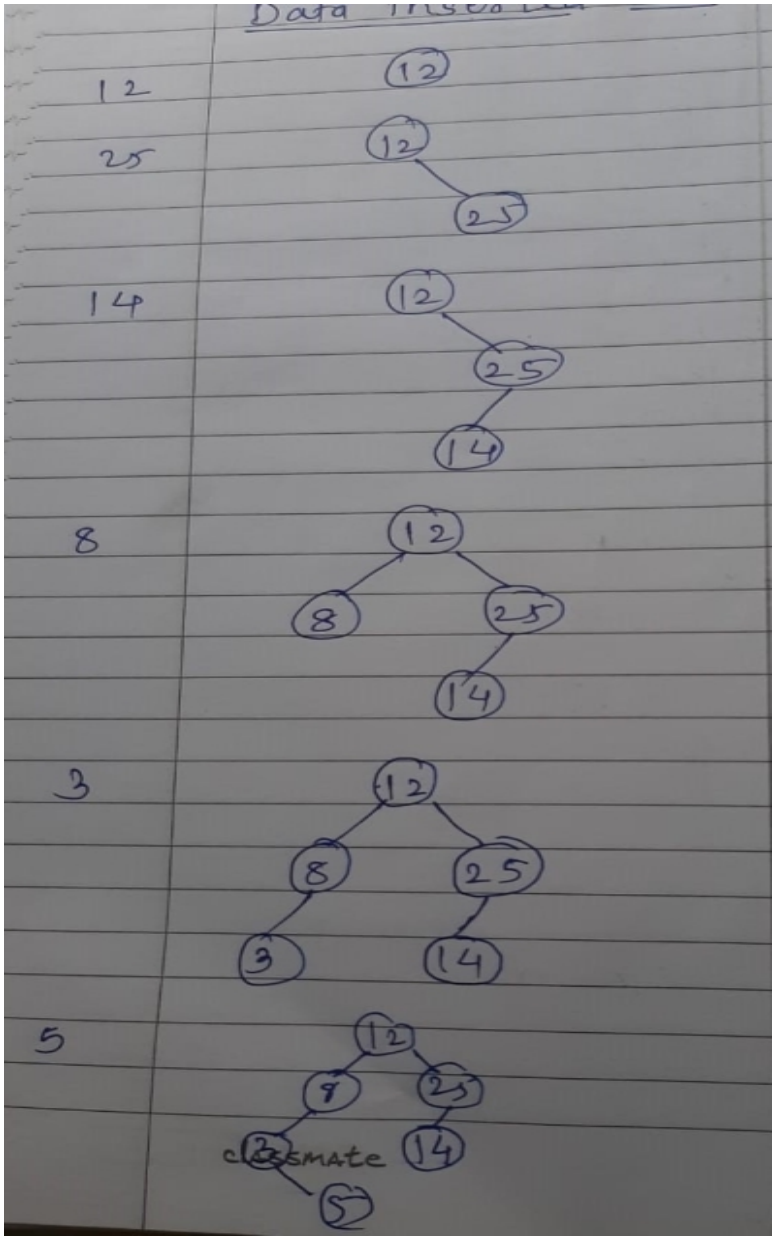
Create a Binary Search tree 50,80,30,20,100,75,25,15



Create a Binary Search tree
J,R,DG,T,E



Create a Binary Search tree
12,25,14,8,3,5



6.5 Binary Tree Traversal

- Binary tree traversing is a process of accessing every node of the tree and exactly once. A tree is defined in a recursive manner. Binary tree traversal also defined recursively.

There are three techniques of traversal:

1. Preorder Traversal **(NLR)**
2. Postorder Traversal**(LRN)**
3. Inorder Traversal**(LNR)**

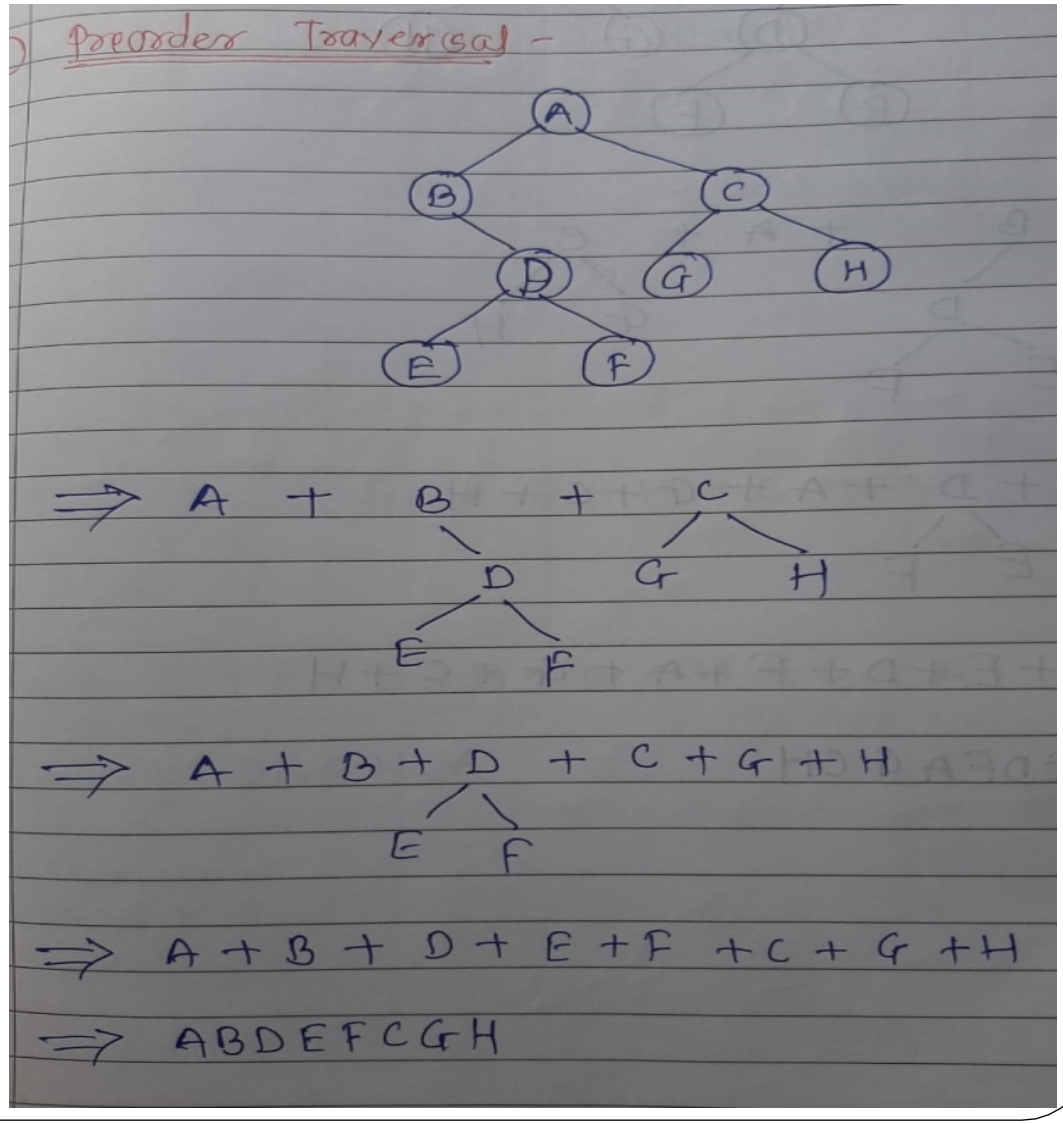
1. Preorder Traversal(NLR)

- **Algorithm for preorder traversal**

Step 1 : Start from the Root.

Step 2 : Then, go to the Left Subtree.

Step 3 : Then, go to the Right Subtree.



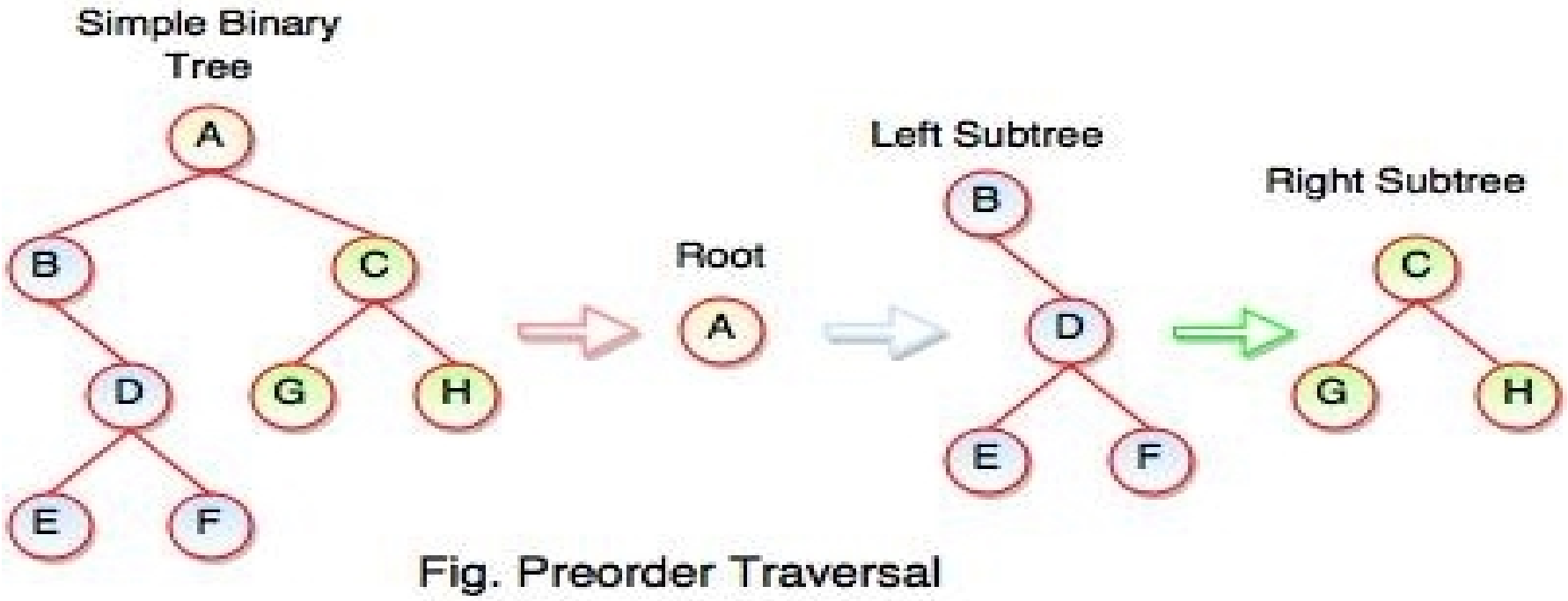


Fig. Preorder Traversal

Step 1 : A + B (B + Preorder on D (D + Preorder on E and F)) + C (C + Preorder on G and H)

Step 2 : A + B + D (E + F) + C (G + H)

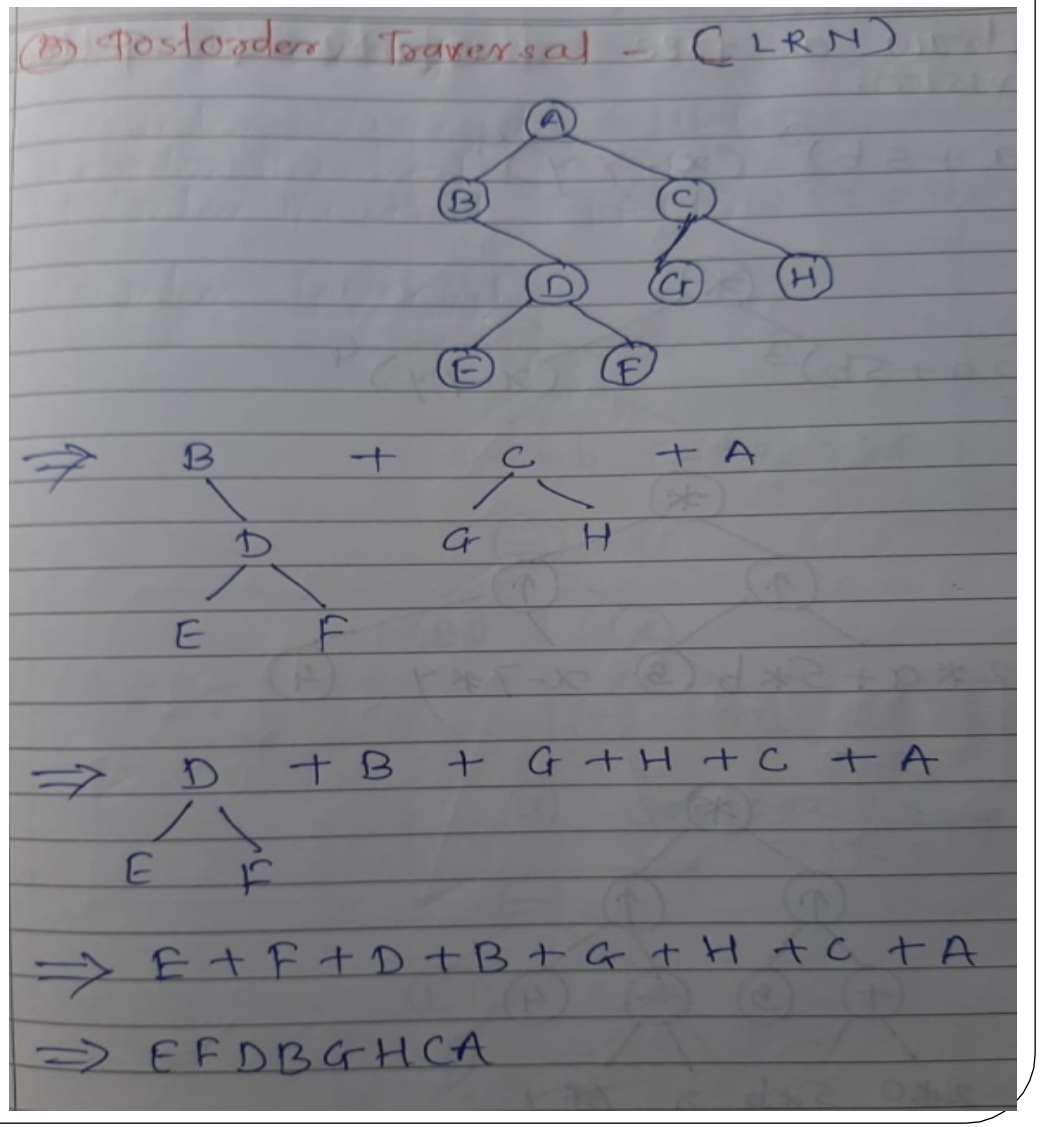
Step 3 : A + B + D + E + F + C + G + H

Preorder Traversal : A B C D E F G H

2. Postorder Traversal(LRN)

Algorithm for postorder traversal :

Step 1 : Start from the Left Subtree (Last Leaf).
Step 2 : Then, go to the Right Subtree.
Step 3 : Then, go to the Root.



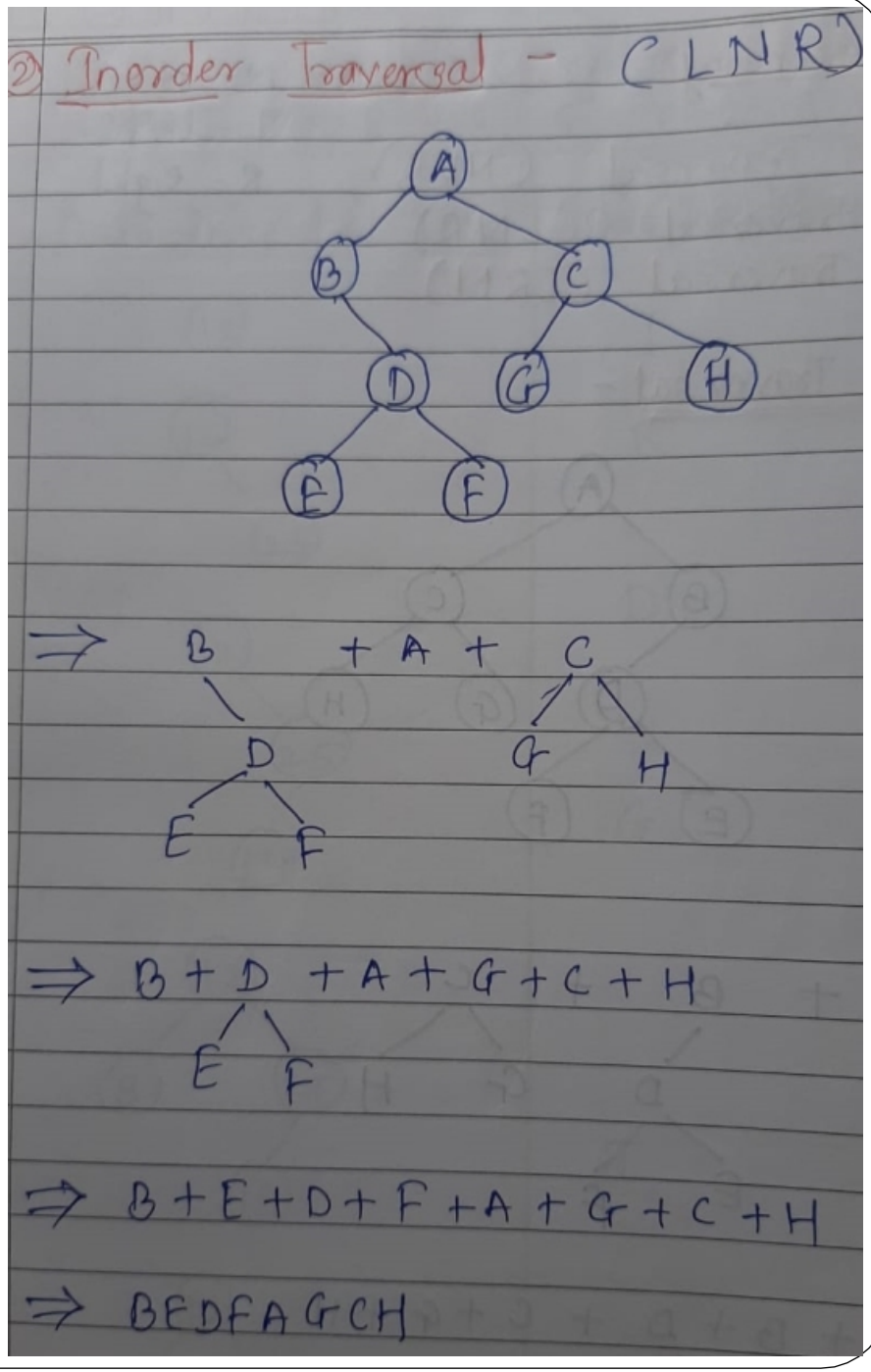
3. Inorder Traversal

Algorithm for inorder traversal

Step 1 : Start from the Left Subtree.

Step 2 : Then, visit the Root.

Step 3 : Then, go to the Right Subtree.



6.7 AVL Tree

AVL-Tree (Balanced Tree)

DATE

"Addson, Velski & Landis (AVL) in 1962 introduced a tree structure that is balanced with respect to the height of the subtrees. such a tree is called as AVL Tree.

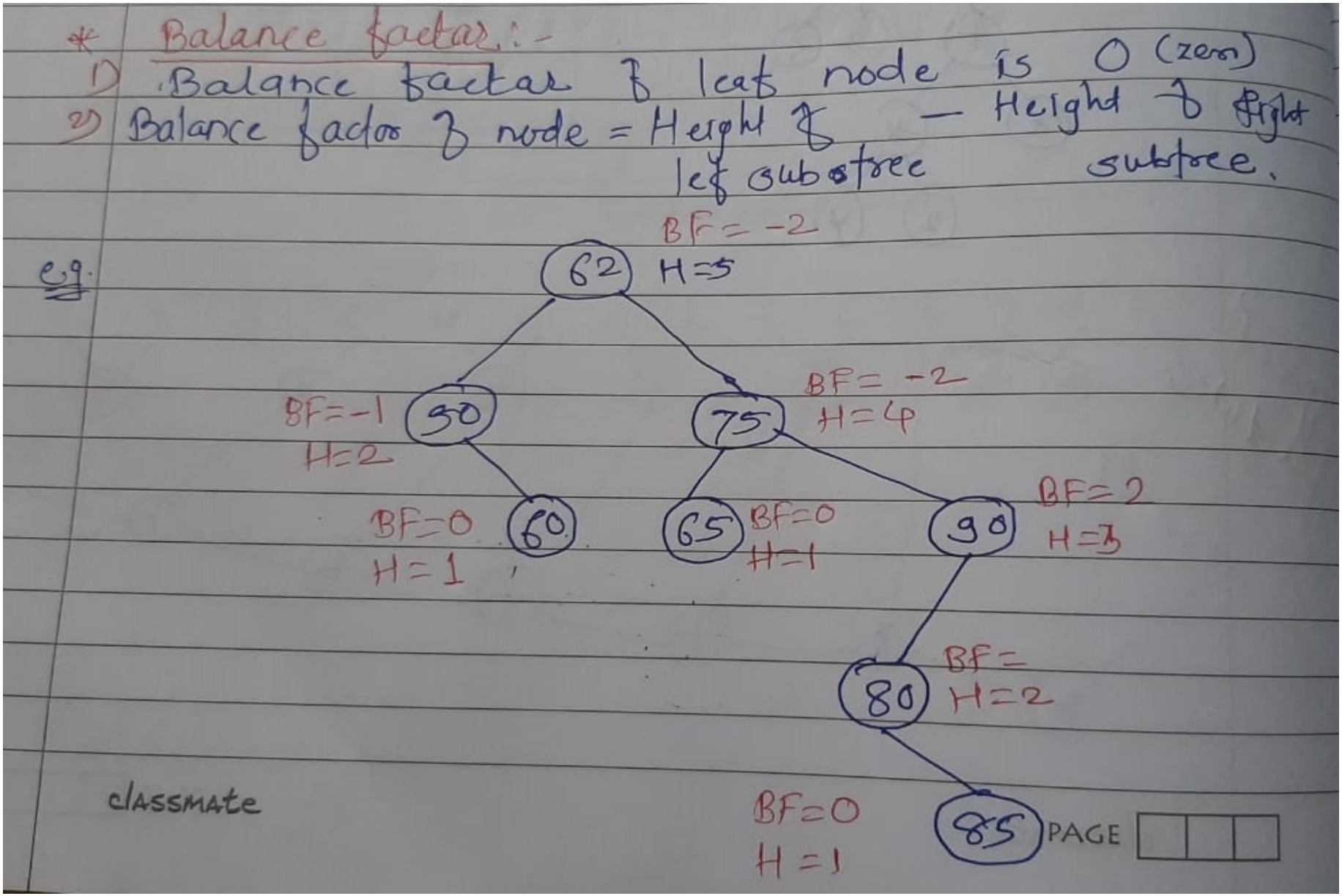
An AVL Tree is binary search tree where height of left and right subtree of any node will be with maximum difference 1.

* **Height of Node :-**

1) Height of leaf node is always 1

2) Height of Internal node = 1 + max [left child, Right child]
(Height)

Balance Factor



Height of Node

1) Height of leaf node is 1

∴ Height(85), Height(65), Height(60) = 1

2) Height of (80) = 1 + max [left child, Height(85)]
= 1 + max [-, 1]
= 1 + 1
= 2

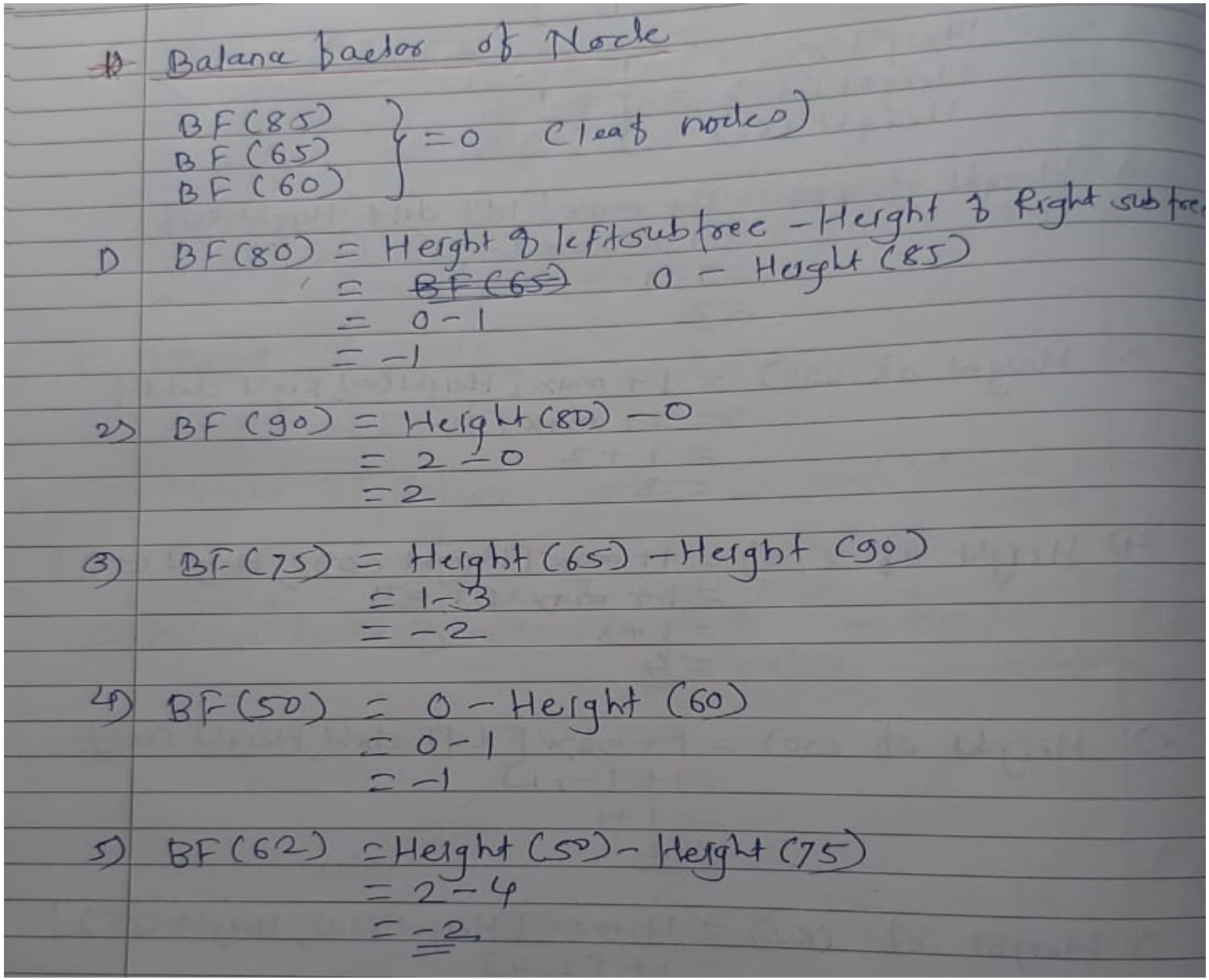
3) Height of (90) = 1 + max [Height(80), Right child]
= 1 + max [2, -]
= 1 + 2
= 3

4) Height of (75) = 1 + max [Height(65), Height(90)]
= 1 + max [1, 3]
= 1 + 3
= 4

5) Height of (50) = 1 + max [Left child, Height(60)]
= 1 + [-, 1]
= 1 + 1
= 2

7) Height of (62) = 1 + max [Height(50), Height(75)]
= 1 + [2, 4]
= 1 + 4
= 5

Balance Factor



AVL Rotations

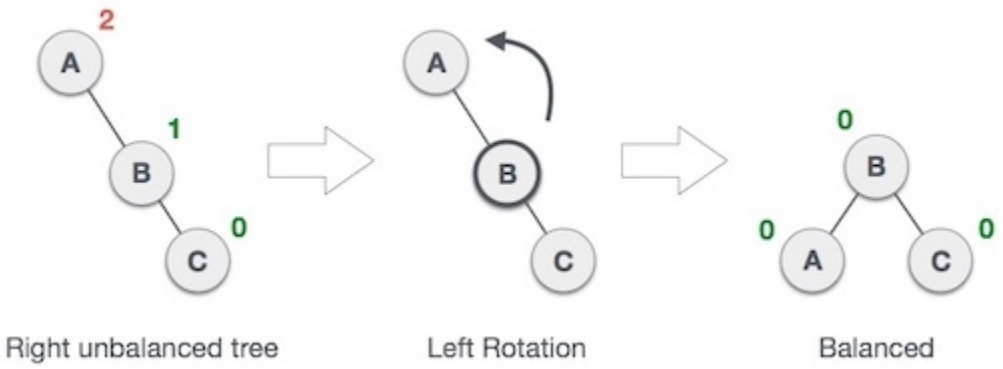
To balance itself, an AVL tree may perform the following four kinds of rotations −

1. Left rotation
2. Right rotation
3. Left-Right rotation
4. Right-Left rotation

The first two rotations are single rotations and the next two rotations are double rotations.

1. Left Rotation

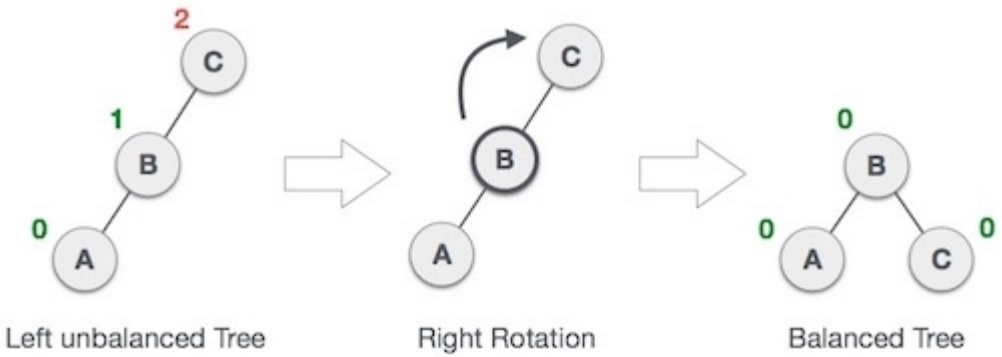
- If a tree becomes unbalanced, when a node is inserted into the right subtree of the right subtree, then we perform a single left rotation −



- In our example, node A has become unbalanced as a node is inserted in the right subtree of A's right subtree. We perform the left rotation by making A the left-subtree of B.

2.Right Rotation

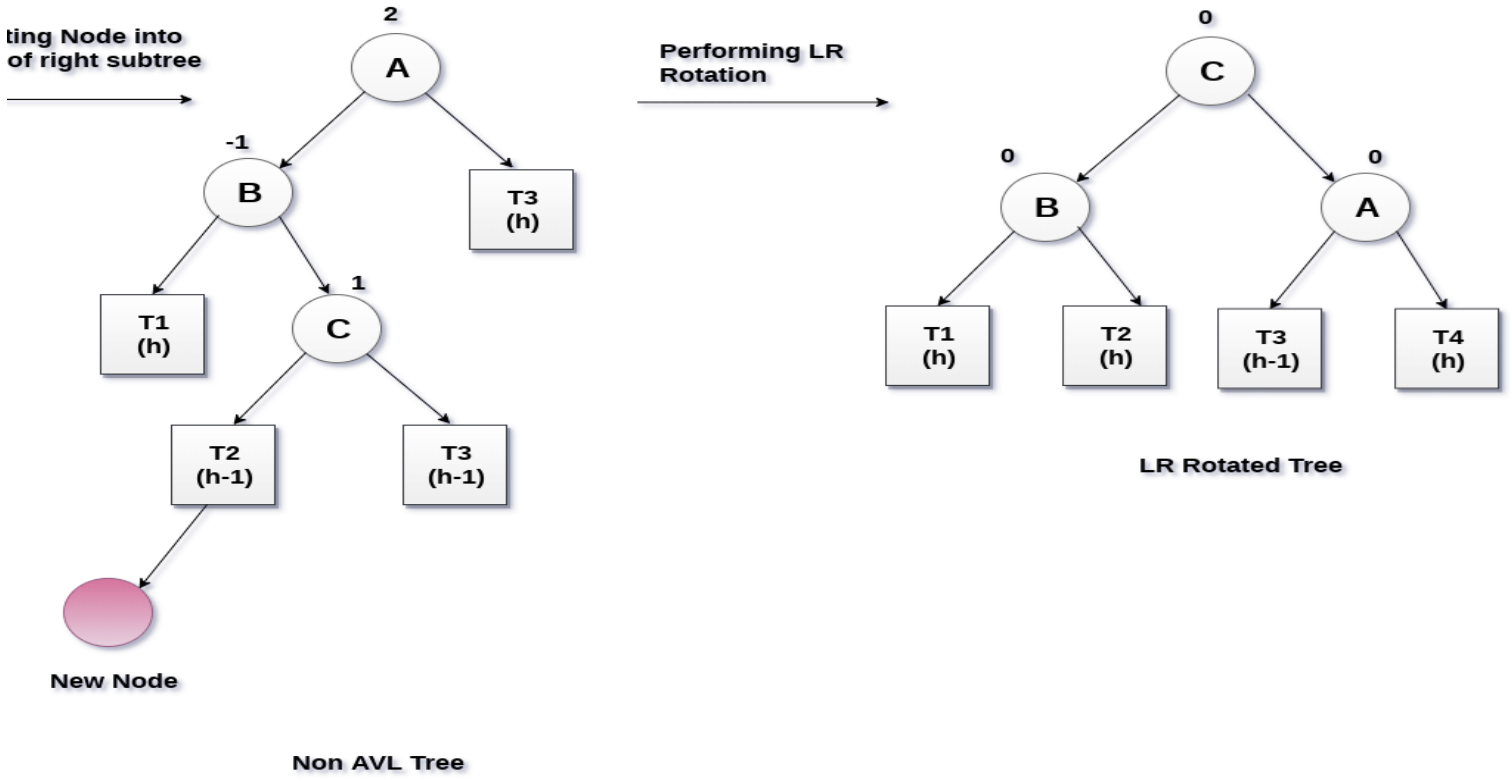
- AVL tree may become unbalanced, if a node is inserted in the left subtree of the left subtree. The tree then needs a right rotation.



As depicted, the unbalanced node becomes the right child of its left child by performing a right rotation.

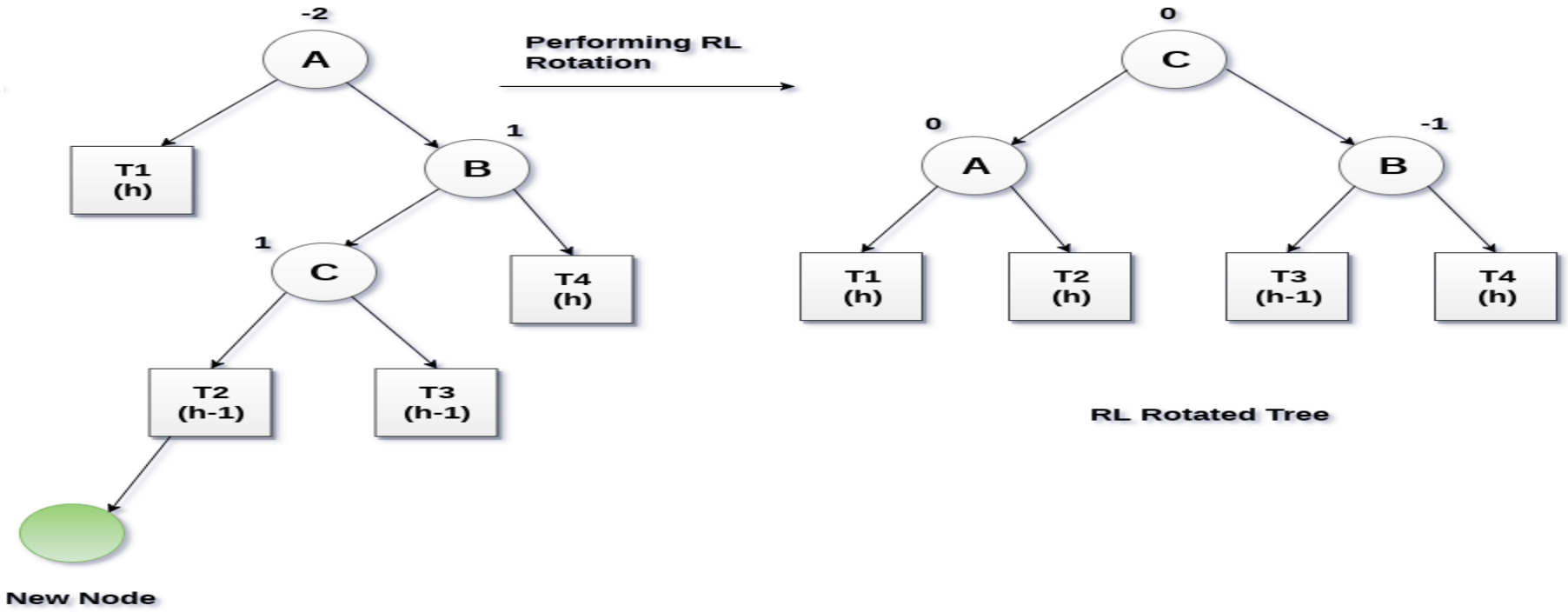
3. Left Right Rotation (LR Rotation)

- The LR Rotation is a sequence of single left rotation followed by a single right rotation. In LR Rotation, at first, every node moves one position to the left and one position to right from the current position. To understand LR Rotation, let us consider the following insertion operation in AVL Tree...

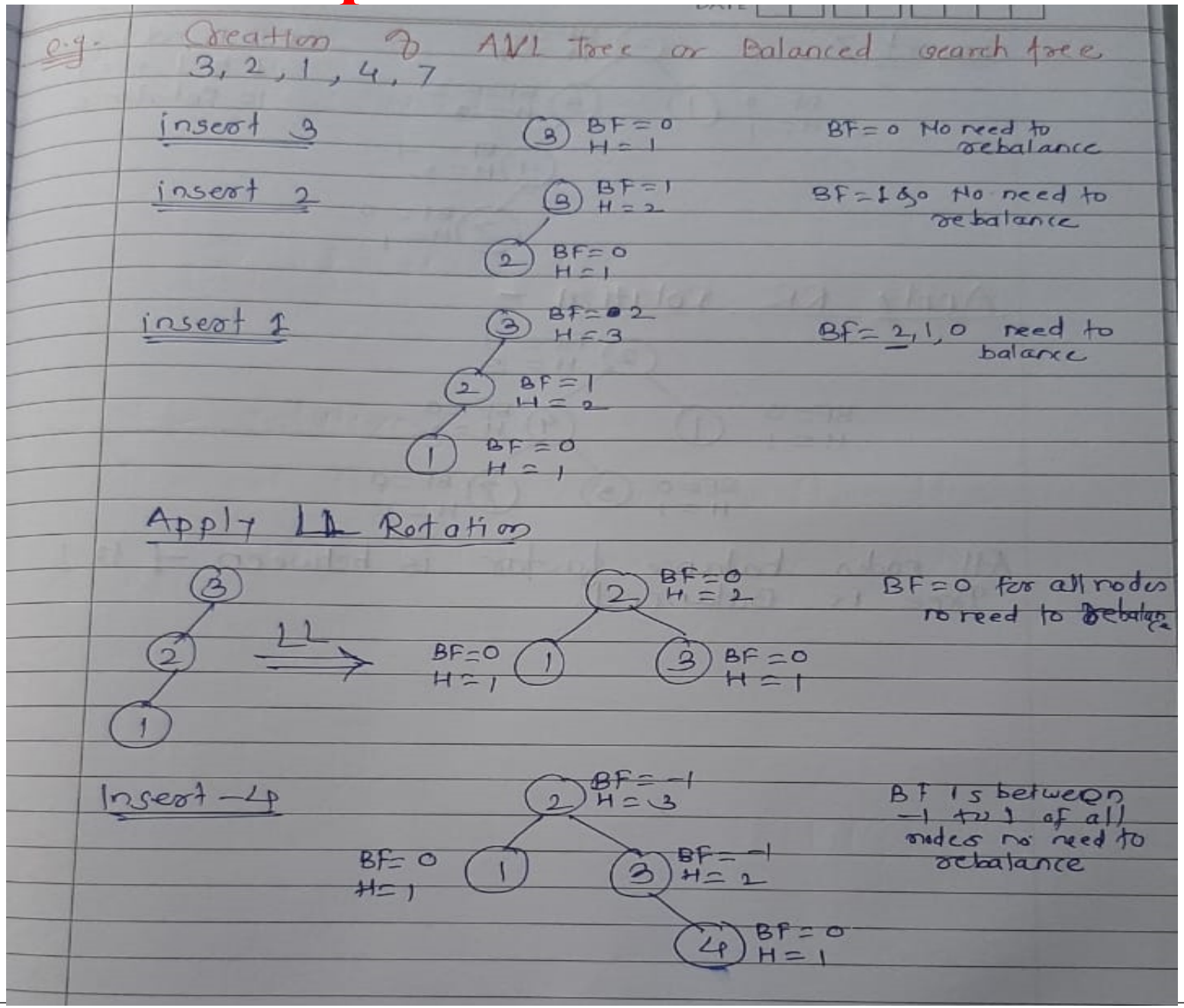


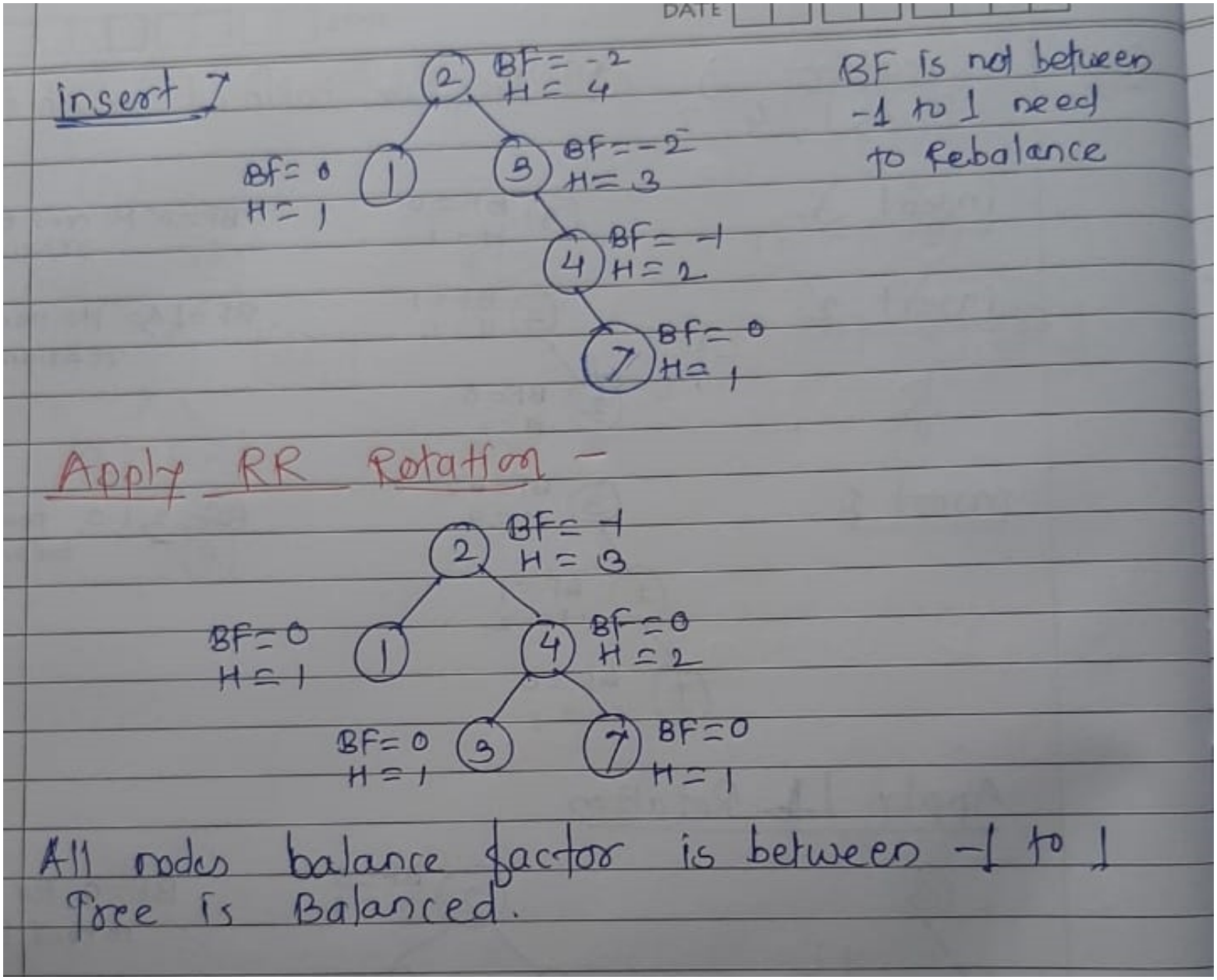
4. Right Left Rotation (RL Rotation)

- The RL Rotation is sequence of single right rotation followed by single left rotation. In RL Rotation, at first every node moves one position to right and one position to left from the current position. To understand RL Rotation, let us consider the following insertion operation in AVL Tree...



Example : To create AVL Tree




insert 7
2
BF = -2
H = 4
BF is not between -1 to 1 need to Rebalance
1
BF = 0
H = 1
3
BF = -2
H = 3
4
BF = -1
H = 2
7
BF = 0
H = 1
Apply RR Rotation -
2
BF = -1
H = 3
1
BF = 0
H = 1
4
BF = 0
H = 2
3
BF = 0
H = 1
7
BF = 0
H = 1
All nodes balance factor is between -1 to 1
Tree is Balanced.

7. Graph

7.1 Concept & Terminology
7.2 Graph Representation-Adjacency Matrix, Adjacency List
7.3Degree of Graph
7.4 Graph Traversal- Breadth First Search (BFS),
Depth First Search (DFS)

7.5 Applications –AOE Network

Unit 6. Graphs

What is Graph?

Graph is a non-linear data structure. It contains a set of points known as nodes (or vertices) and a set of links known as edges (or Arcs). Here edges are used to connect the vertices.

A graph is defined as follows...

Graph is a collection of vertices and arcs in which vertices are connected with arcs

Graph is a collection of nodes and edges in which nodes are connected with edges

Graph consists of two following components:

1. Vertices
2. Edges

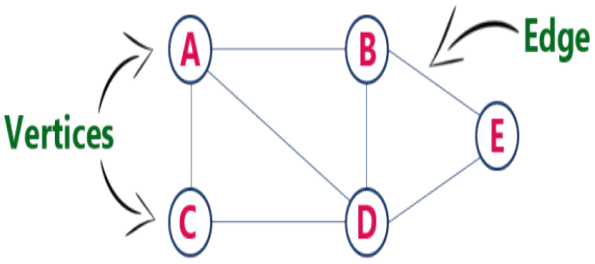
- Graph is a set of vertices (V) and set of edges (E).
- V is a finite number of vertices also called as nodes.
- E is a set of ordered pair of vertices representing edges.
- Generally, a graph **G** is represented as **G = (V , E)**, where **V is set of vertices** and **E is set of edges**.

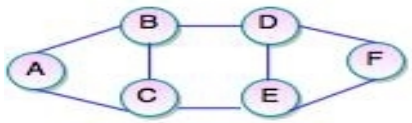
Example The following is a graph with 5 vertices and 6 edges.

This graph G can be defined as G = (V , E)

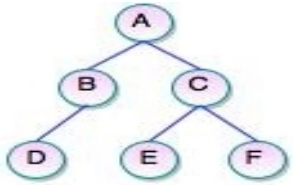
Where V = {A,B,C,D,E}

E = {(A,B),(A,C)(A,D),(B,D),(C,D),(B,E),(E,D)}.

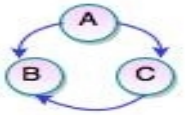




Graph 1



Graph 2



Graph 3

Fig. Graphs

Types of Graph

1. Directed Graph
2. Undirected Graph
3. Connected Graph
4. Disconnected Graph
5. Mixed Type Graph
6. Cyclic Graph
7. Acyclic Graph
8. Weighted Graph

1. Directed Graph :

A graph with only directed edges is called directed graph.

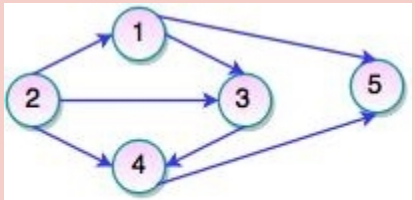


Fig. Directed Graph

2. Undirected Graph :

A graph without direction is called undirected graph.

In which each edge is not assigned a direction.

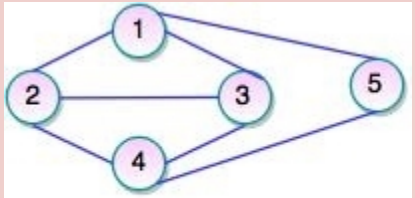
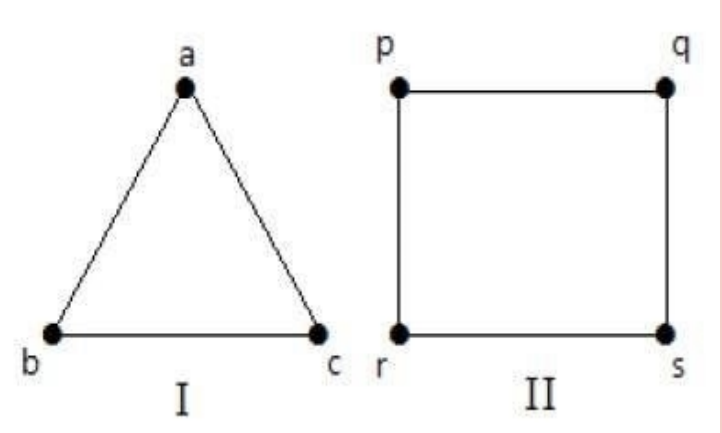


Fig. Undirected Graph

3. Connected Graph

A graph G is said to be connected **if there exists a path between every pair of vertices**. There should be at least one edge for every vertex in the graph. So that we can say that it is connected to some other vertex at the other side of the edge.

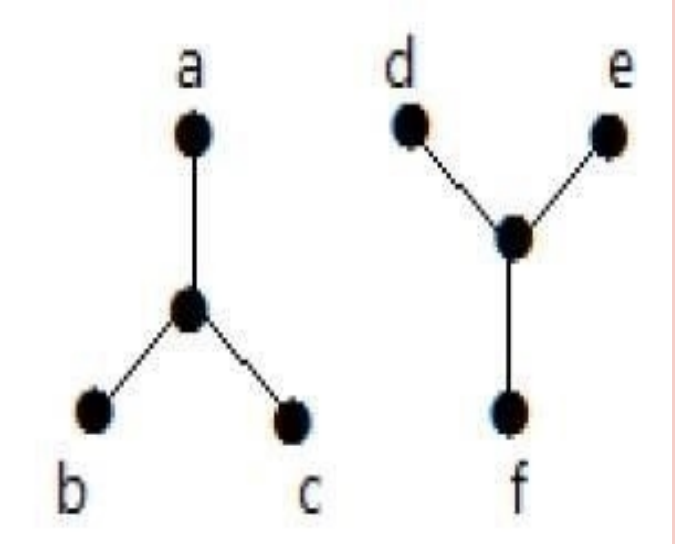


4. Disconnected Graph :

A graph G is disconnected,
if it does not contain at least two connected vertices.

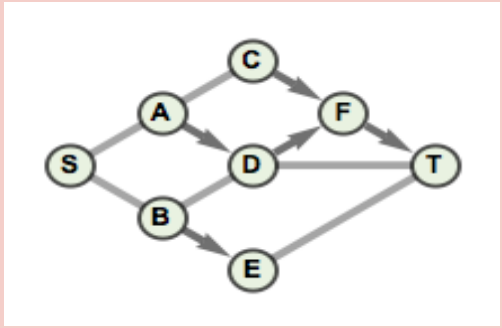
The following graph is an example of a Disconnected Graph, where there are two components, one with 'a', 'b', 'c' vertices and another with 'e', 'f', 'g'vertices.

The two components are independent and not connected to each other. Hence it is called disconnected graph.



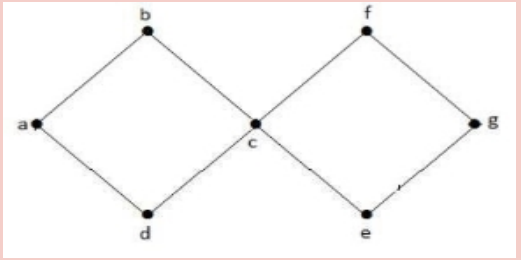
5. Mixed Type Graph

A graph in which some edges are directed and some edges are undirected such a graph is called as mixed type graph.



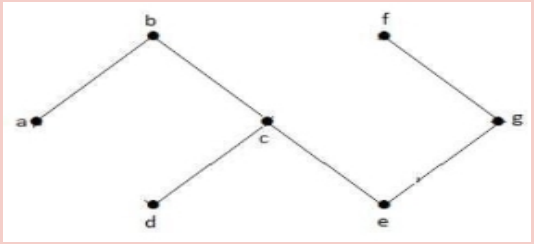
6. Cyclic Graph :

A graph **with at least one** cycle is called a cyclic graph.



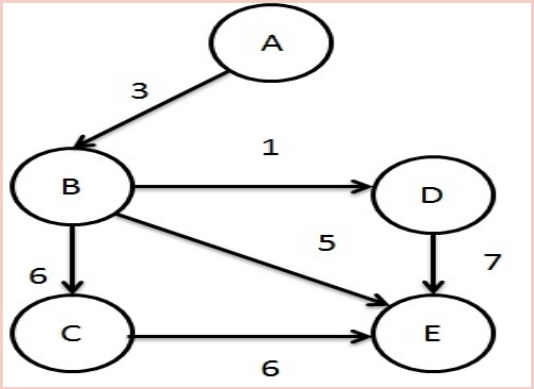
7. Acyclic Graph :

A graph **with no** cycle is called Acyclic graph.



8. Weighted Graph :

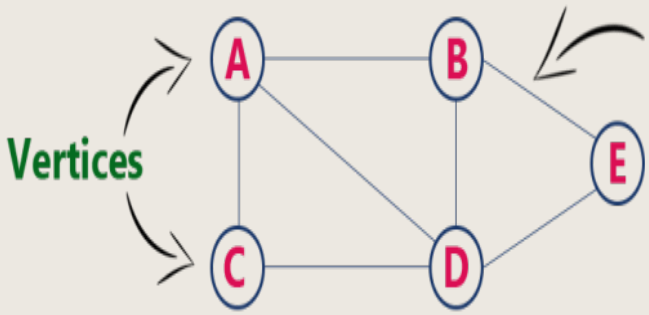
In a weighted graph, each edge has an associated numerical value, called the weight of the edge
Edge weights may represent distance, time, cost, etc.
A weighted graph can be directed or undirected.



Graph - Terminology

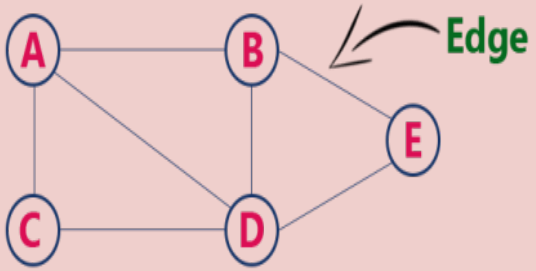
1. Vertex

Individual data element of a graph is called as Vertex. **Vertex** is also known as **node**.
In above example graph, A, B, C, D & E are known as vertices.



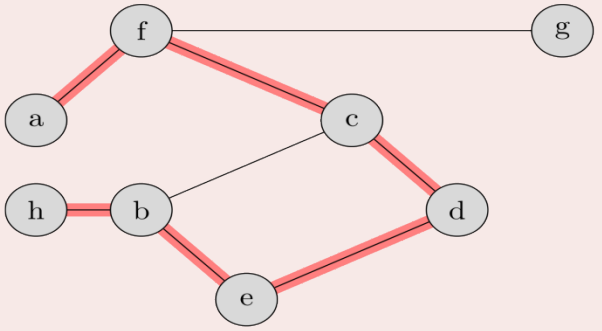
2. Edge

An edge is a connecting link between two vertices.
Edge is also known as **Arc**.



3. Path

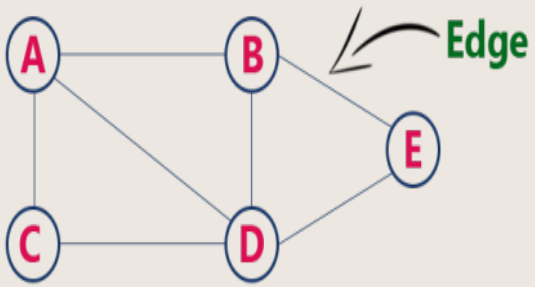
A path is a sequence of alternate vertices and edges that starts at a vertex and ends at other vertex such that each edge is incident to its predecessor and successor vertex.



Graph - Terminology

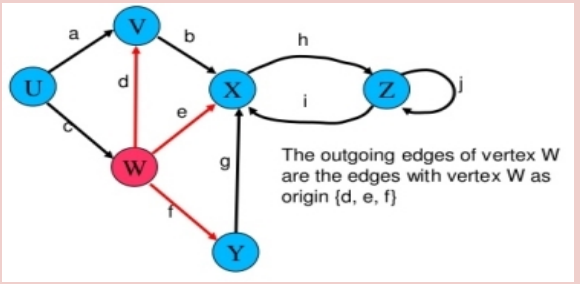
4. Adjacent node

Vertices B and E are said to be adjacent node .
If there is an edge between vertices B and E



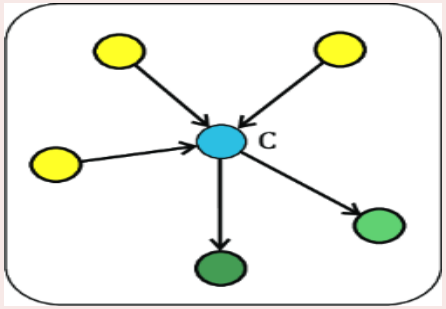
5. Outgoing Edge

A directed edge is said to be outgoing edge on its origin vertex.



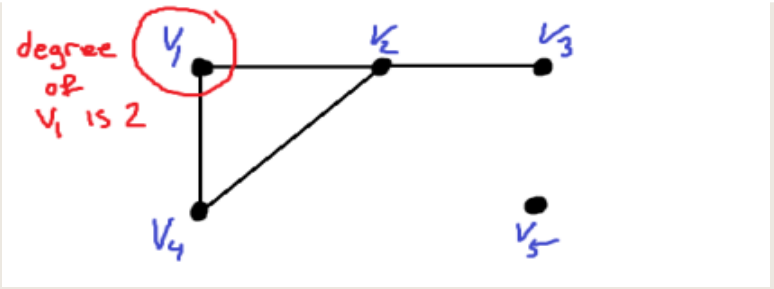
6. Incoming Edge

A directed edge is said to be incoming edge on its destination vertex.



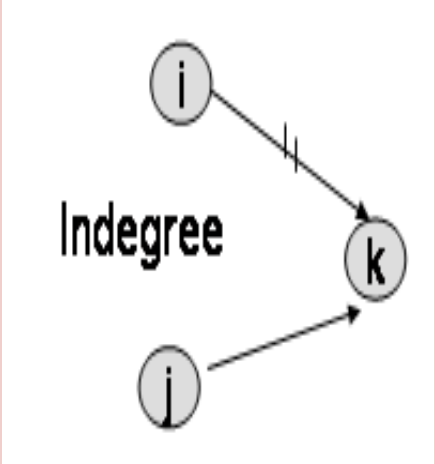
7. Degree

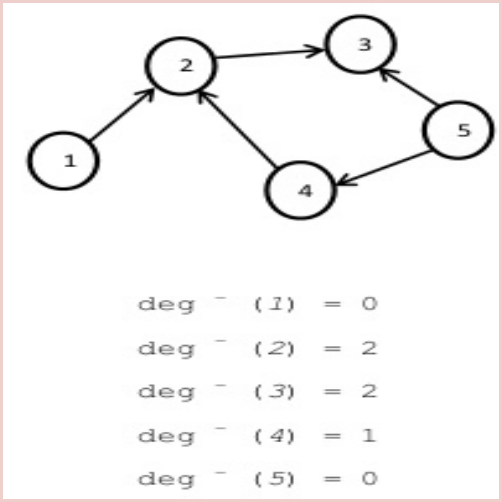
Total number of edges connected to a vertex is said to be degree of that vertex.



8. In degree

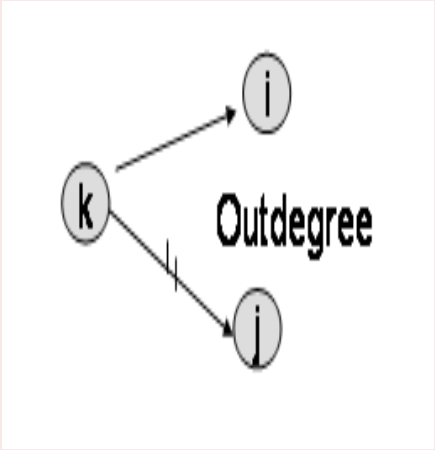
Total number of incoming edges connected to a vertex is said to be in degree of that vertex.

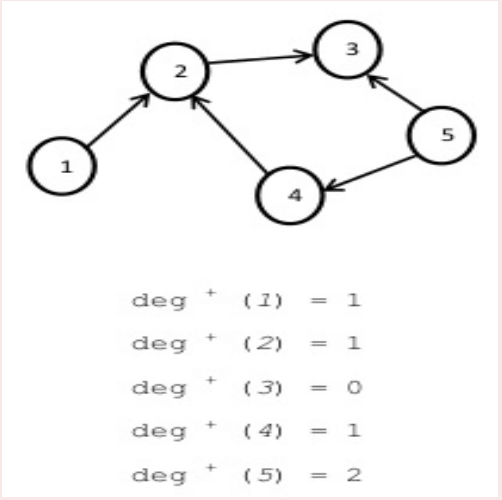




9. Out degree

Total number of outgoing edges connected to a vertex is said to be out degree of that vertex.

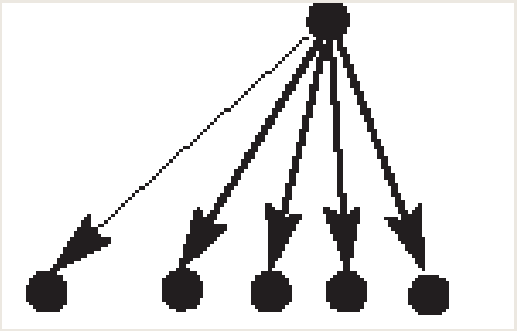




Graph - Terminology

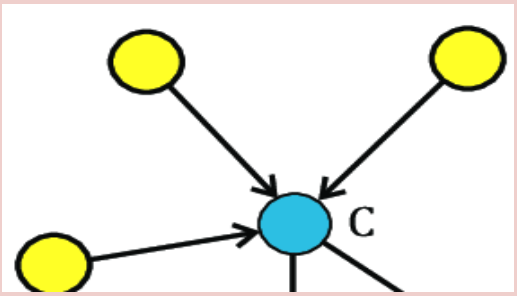
10. Source

A node which has only out going edges and no incoming edges is called source



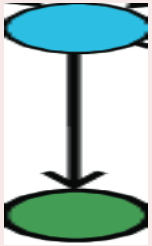
11. Sink

A node which has only incoming edges and no outgoing edges is called sink



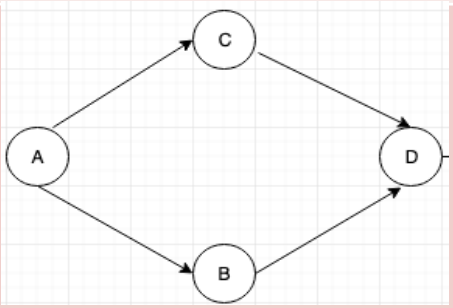
12. Pendant Node

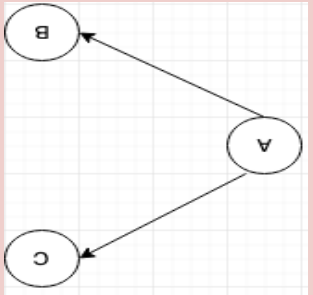
When indegree of node is one and out degree is zero then such a node is called pendant node vertex.



13. Articulation Point

If on removing the node the graph gets disconnected then that node is called articulation point..

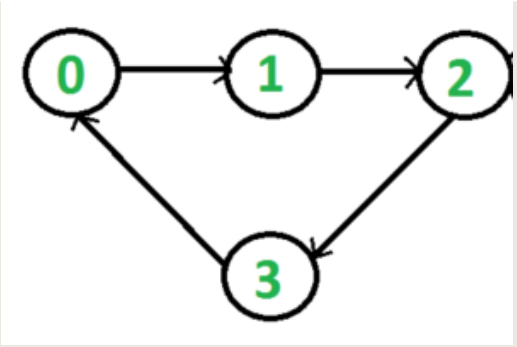




Graph - Terminology

14. Cycle

A path from node to itself is called cycle.
Thus cycle is a path in which the initial and final vertex is same

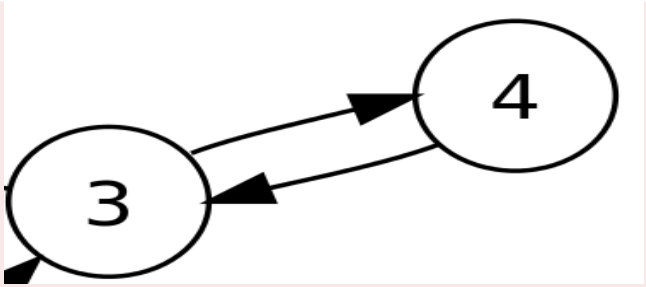


15. Sling or loop

An edge of a graph which joins a node to itself is called a sling or loop

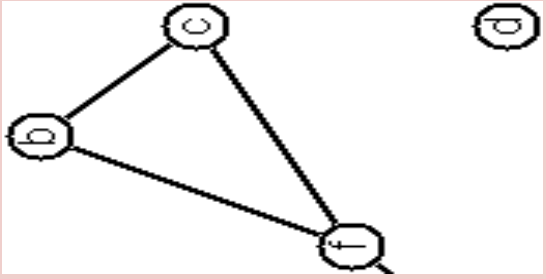
16. Parallel Edges

The two distinct edges between a pair to nodes which are opposite in direction are called as parallel edges.



17. Isolated node

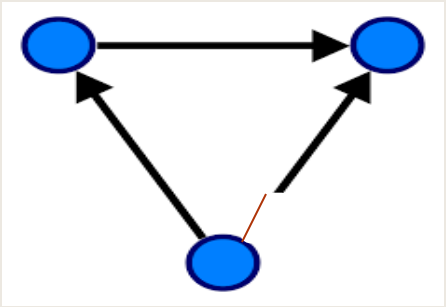
A node which is not an adjacent neighbor to any other node is called an isolated node.



Graph - Terminology

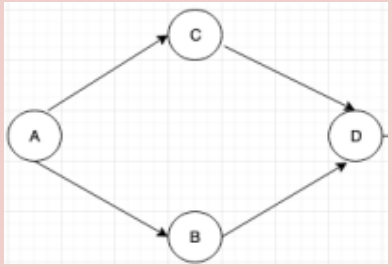
18. Directed Acyclic Graph

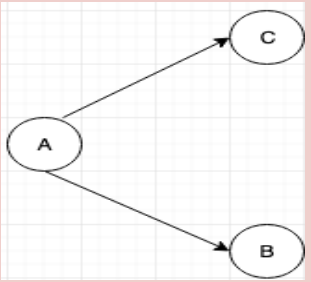
A directed graph with no cycle is called directed acyclic graph.



19. Sub graph

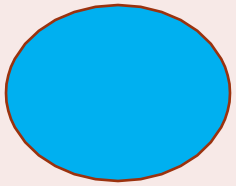
Sub graph is a graph G





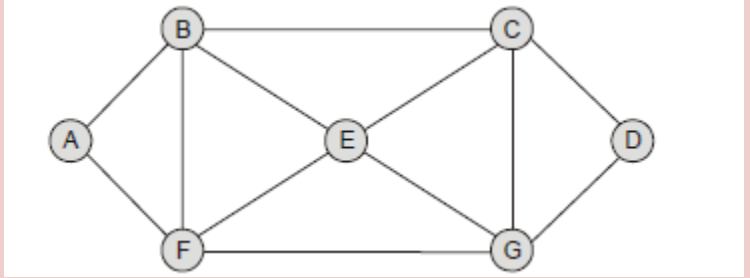
20. Isolated / Null Graph

A graph which has set of empty edges or is containing only isolated nodes is called NULL graph or or isolated graph



21. Biconnected Graph

A Biconnected graph is the graph which does not contain any articulation point.



Graph Representations

Graph data structure is represented using following representations…

1. **Adjacency Matrix**
2. **Adjacency List**

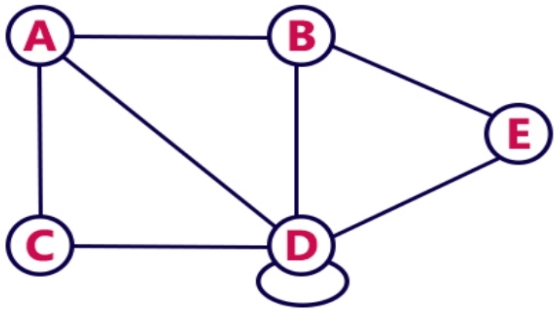
1. Adjacency Matrix

In this representation, the graph is represented using a matrix of size total number of vertices. That means a graph with 4 vertices is represented using a matrix of size 4X4.

In this matrix, both rows and columns represent vertices.

This matrix is filled with either 1 or 0. Here, 1 represents that there is an edge from row vertex to column vertex and 0 represents that there is no edge from row vertex to column vertex.

For example, consider the following undirected graph representation…



	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	1	0
D	1	1	1	1	1
E	0	1	0	1	0

Degree of Nodes

A=3

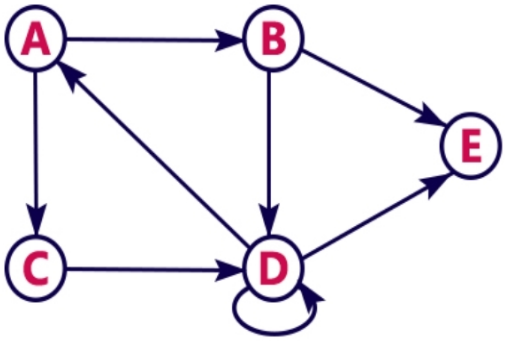
B=3

C=2

D=5

E=2

Directed graph representation...



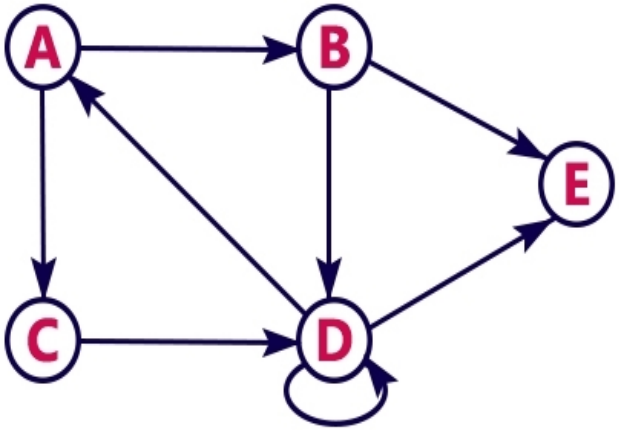
	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	1
C	0	0	0	1	0
D	1	0	0	1	1
E	0	0	0	0	0

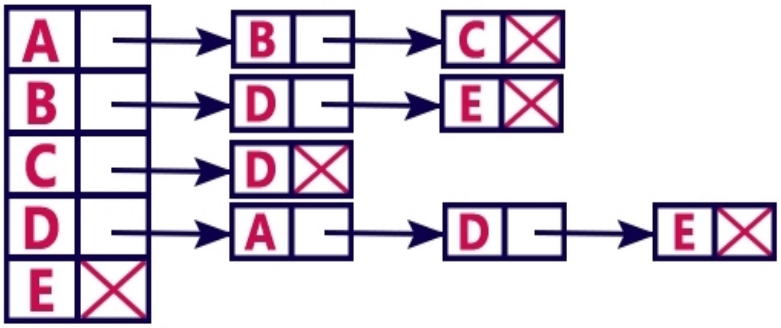
InDegree of Nodes	OutDegree of Nodes
A=1	A=2
B=1	B=2
C=1	C=1
D=3	D=3
E=2	E=0

2. Adjacency List

In this representation, every vertex of a graph contains list of its adjacent vertices.

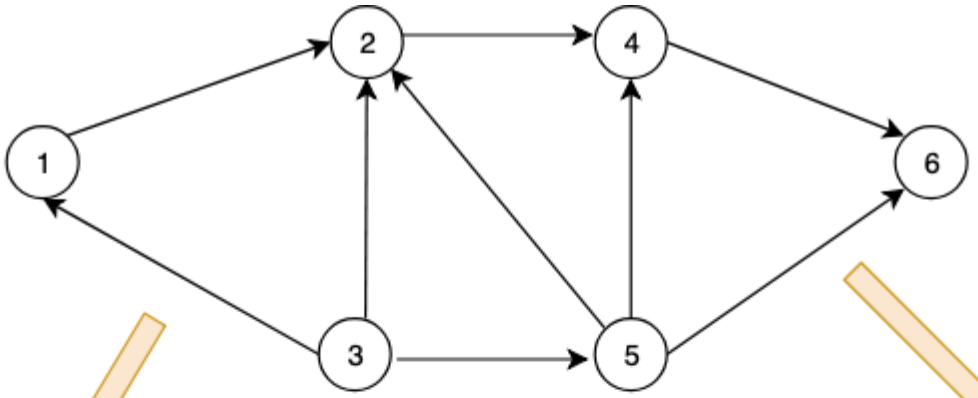
For example, consider the following directed graph representation implemented using linked list…





Example : Graph data structure is represented using following representations…

1. Adjacency Matrix 2. Adjacency List



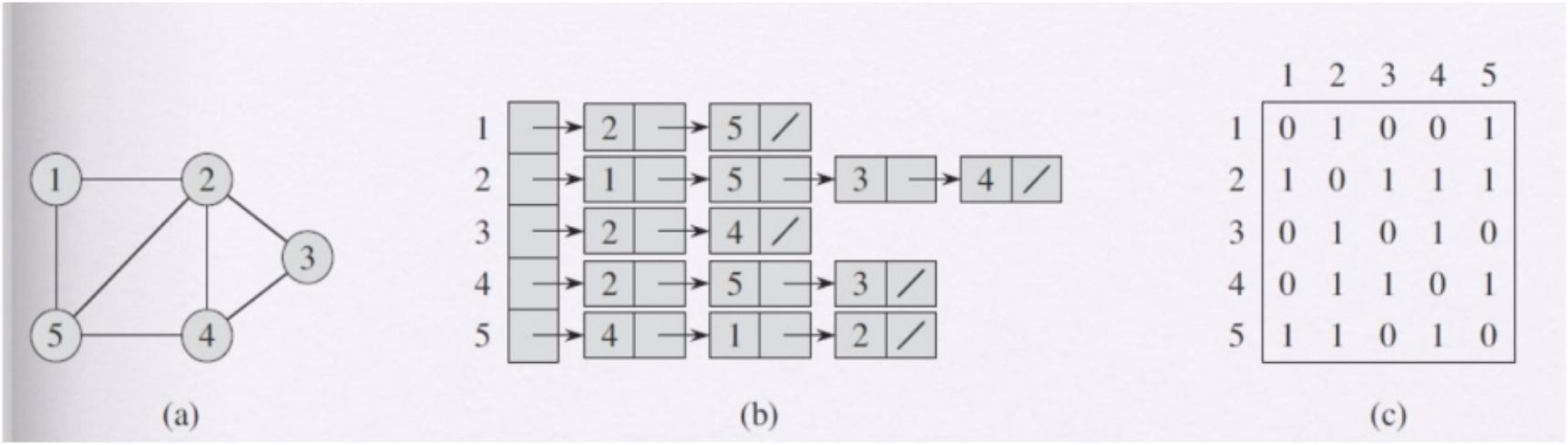
Adjacency List

Adjacency Matrix

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	0	1	0	0
3	1	1	0	0	1	0
4	0	0	0	0	0	1
5	0	1	0	1	0	1
6	0	0	0	0	0	0

Example : Graph data structure is represented using following representations…

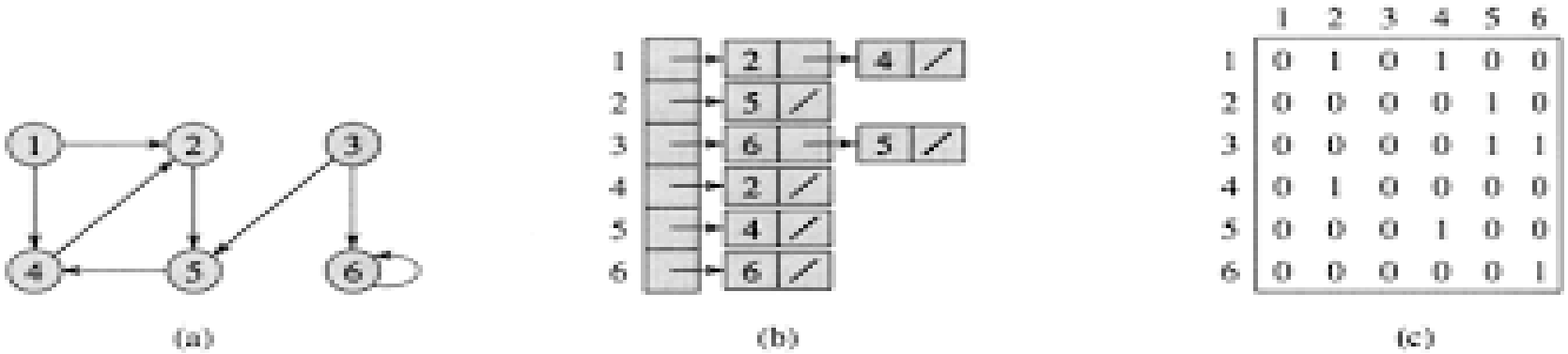
1. Adjacency Matrix 2. Adjacency List



	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Example : Graph data structure is represented using following representations…

1. Adjacency Matrix 2. Adjacency List



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

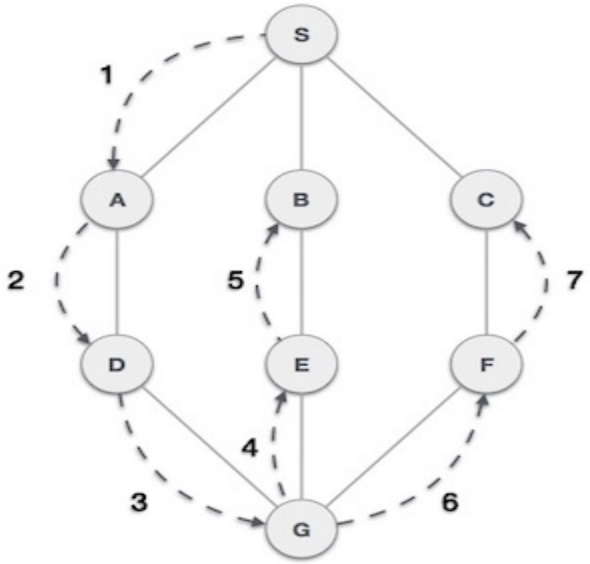
Figure 23.2 Two representations of a directed graph. (a) A directed graph G having six vertices and eight edges. (b) An adjacency-list representation of G. (c) The adjacency-matrix representation of G.

7.4 Graph Traversal

1. Depth First Search (DFS) algorithm
2. BREADTH First Search (BFS) algorithm

1. Depth First Search (DFS) algorithm

Depth First Search (DFS) algorithm traverses a graph in a depth ward motion and uses a stack to remember to get the next vertex to start a search, when a dead end occurs in any iteration.

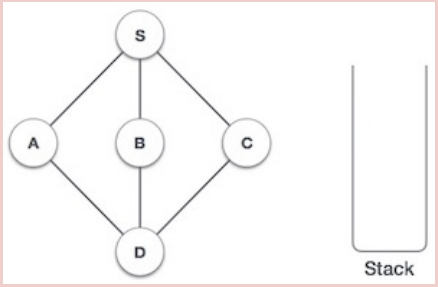
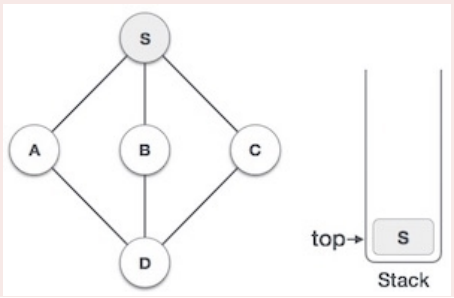
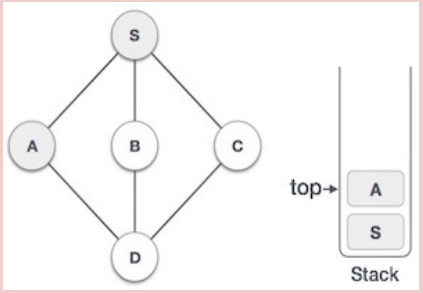
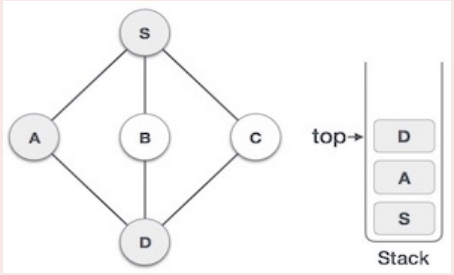


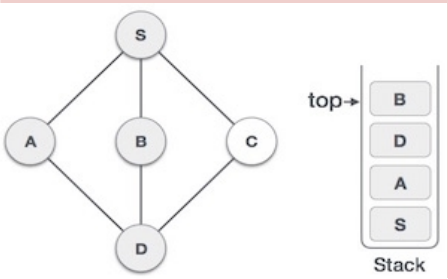
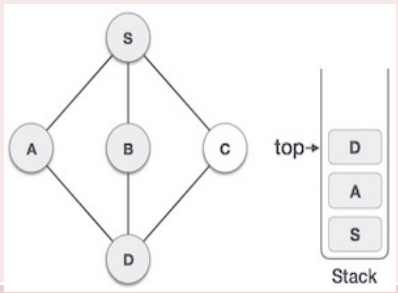
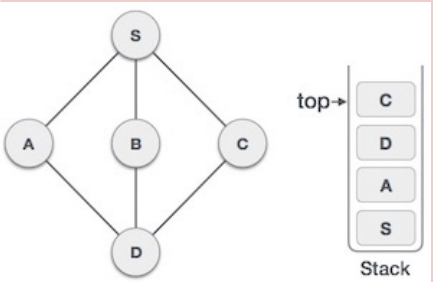
As in the example given above, DFS algorithm traverses from S to A to D to G to E to B first, then to F and lastly to C. It employs the following rules.

Rule 1 – Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.

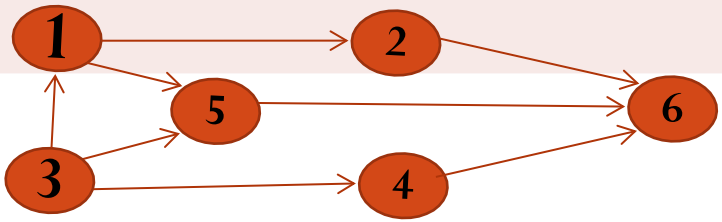
Rule 2 – If no adjacent vertex is found, pop up a vertex from the stack. (It will pop up all the vertices from the stack, which do not have adjacent vertices.)

Rule 3 – Repeat Rule 1 and Rule 2 until the stack is empty.

Step	Traversal	Description
1		Initialize the stack.
2		Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.
3		Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S** and **D** are adjacent to **A** but we are concerned for unvisited nodes only.
4		Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.

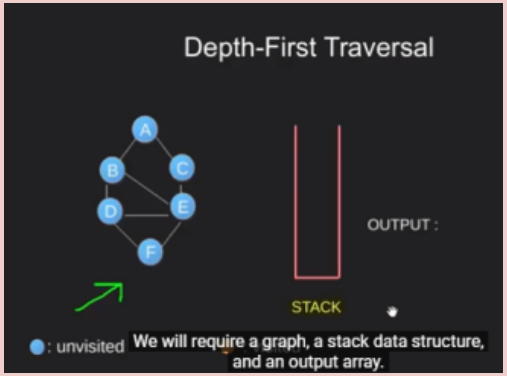
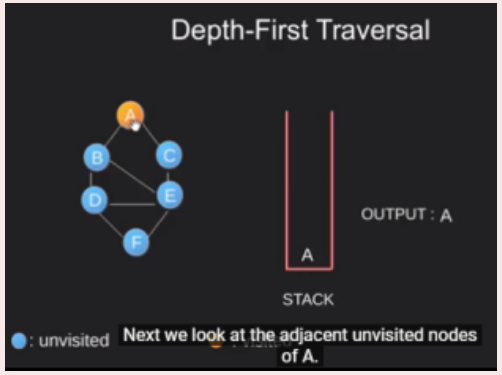
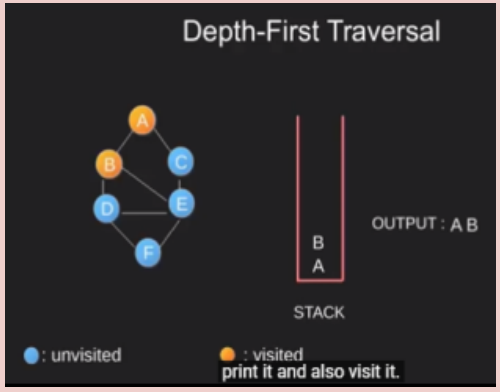
Step	Traversal	Description
5		We choose **B**, mark it as visited and put onto the stack. Here **B** does not have any unvisited adjacent node. So, we pop **B** from the stack.
6		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.
7		Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.

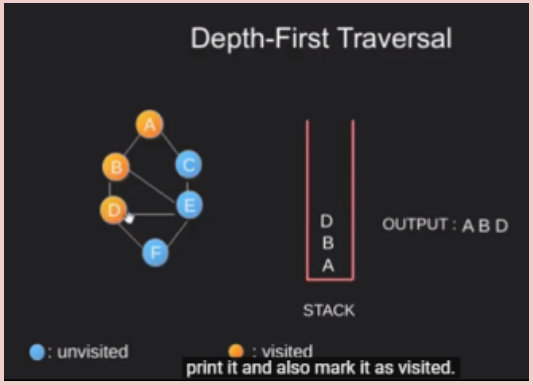
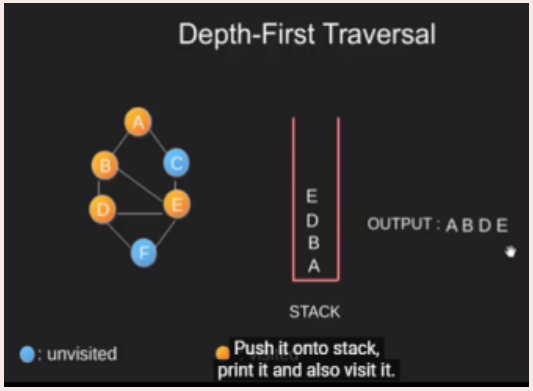
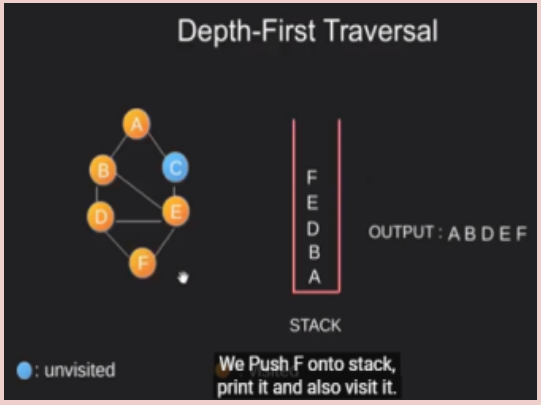
As **C** does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node. In this case, there's none and we keep popping until the stack is empty **DFS Traversing = S,A,D,C**

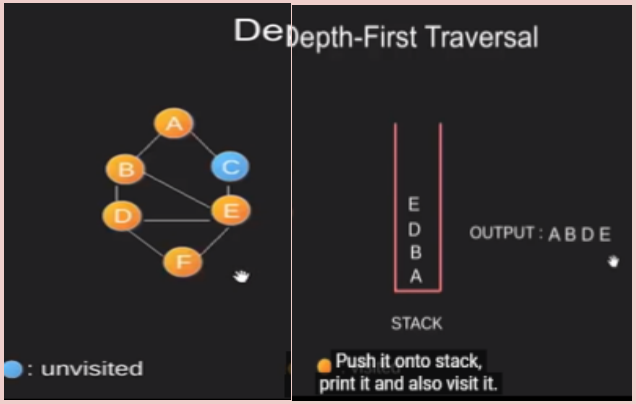
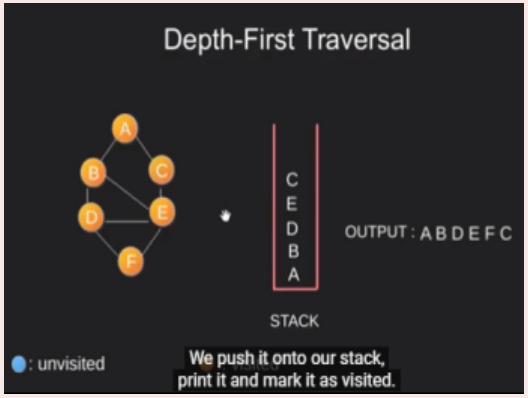


Solve Example DFS

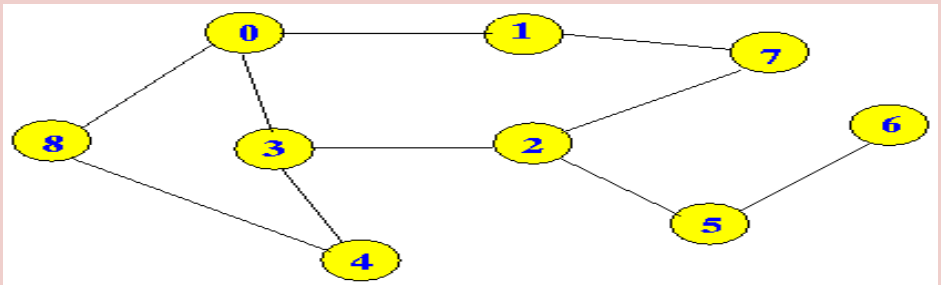
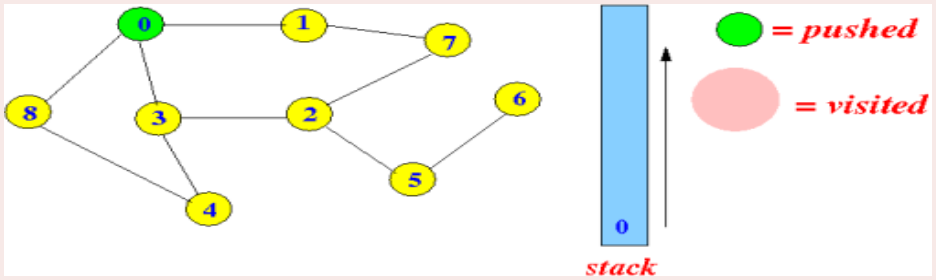
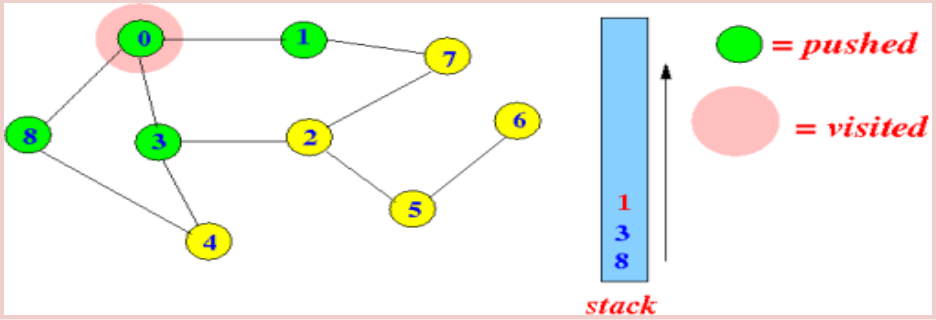
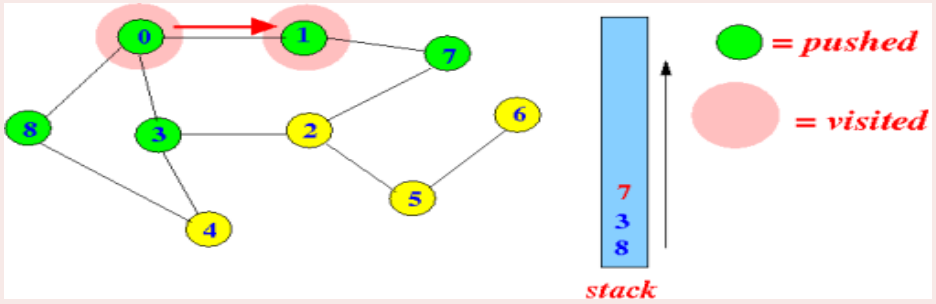
DFS Traversing =

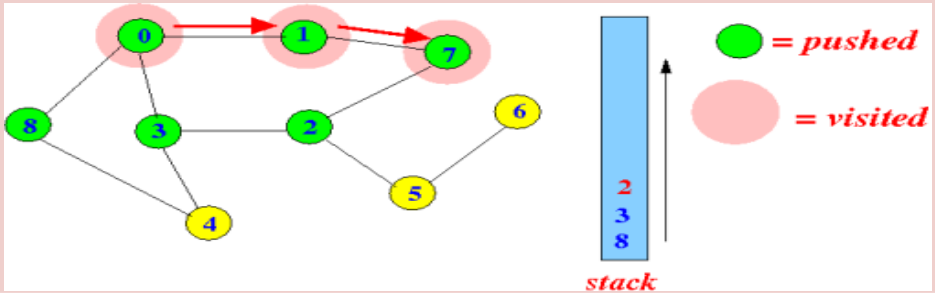
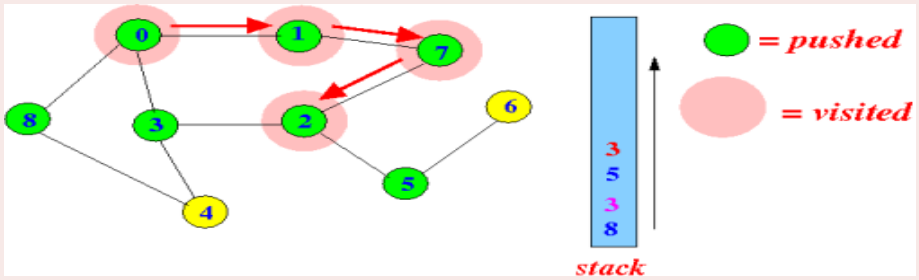
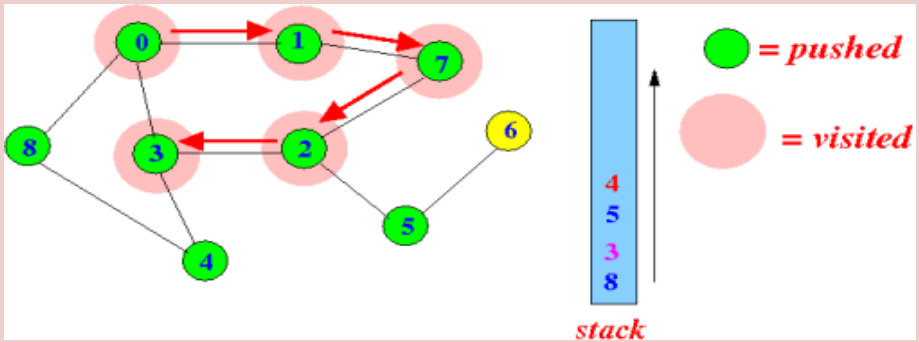
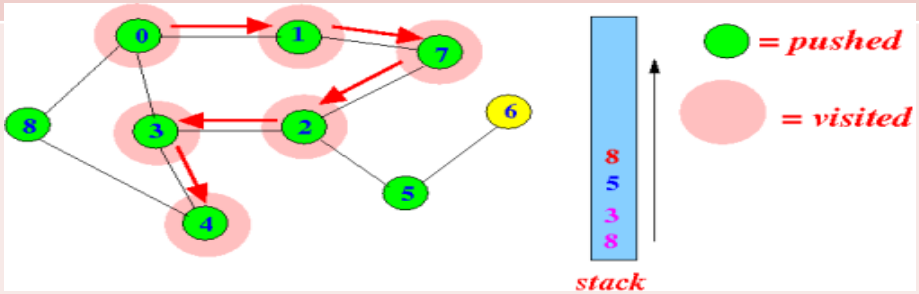
Step	Traversal	Description
1		Initialize the stack.
2		Mark A as visited and put it onto the stack. Explore any unvisited adjacent node from A. We have two nodes (B and C) and we can pick any of them. For this example, we shall take the node in an alphabetical order is B
3		We choose **B**, mark it as visited and put onto the stack. Explore any unvisited adjacent node from B. We have two nodes (D and E) and we can pick any of them. For this example, we shall take the node in an alphabetical order is D

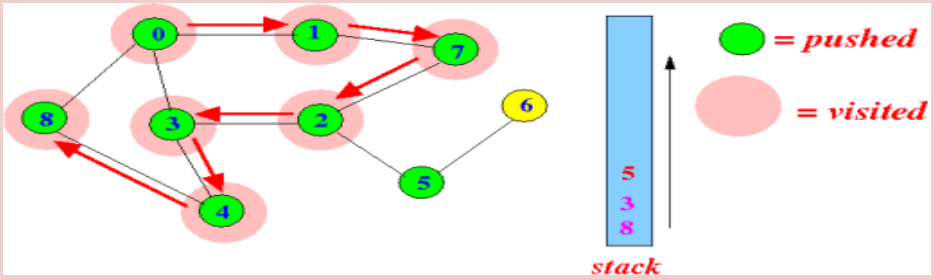
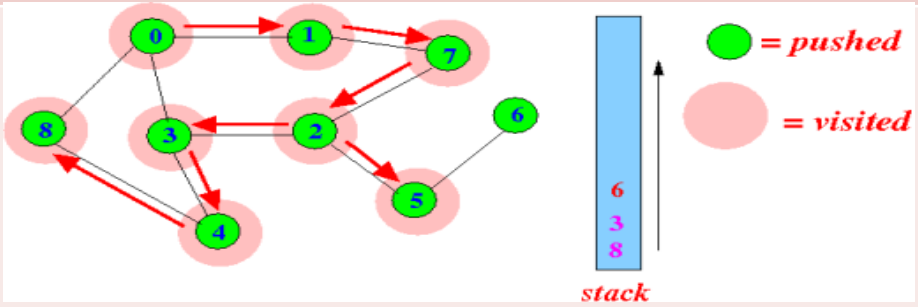
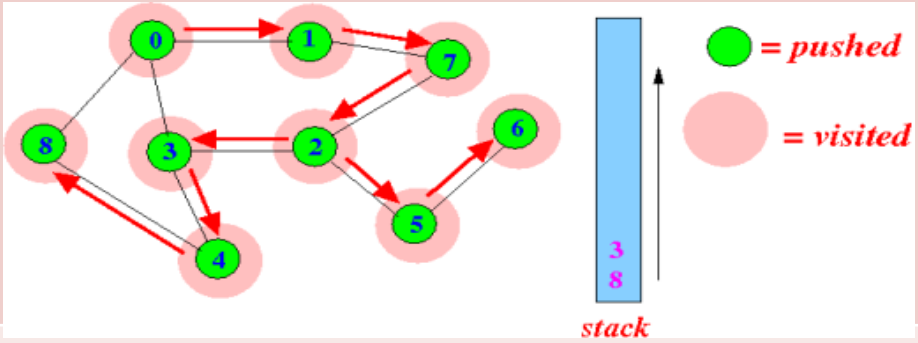
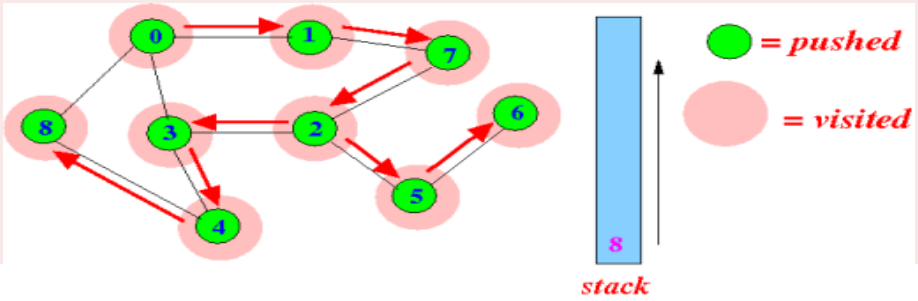
Step	Traversal	Description
5		We choose D, mark it as visited and put onto the stack. Explore any unvisited adjacent node from D . We have two nodes (E and F) and we can pick any of them. For this example, we shall take the node in an alphabetical order is E
6		We choose E, mark it as visited and put onto the stack. Explore any unvisited adjacent node from E . We have two nodes (F and C) and we can pick any of them. For this example, we shall take the node F
7		We choose F, mark it as visited and put onto the stack. Explore any unvisited adjacent node from F . Here **F** does not have any unvisited adjacent node. So, we pop **F** from the stack.

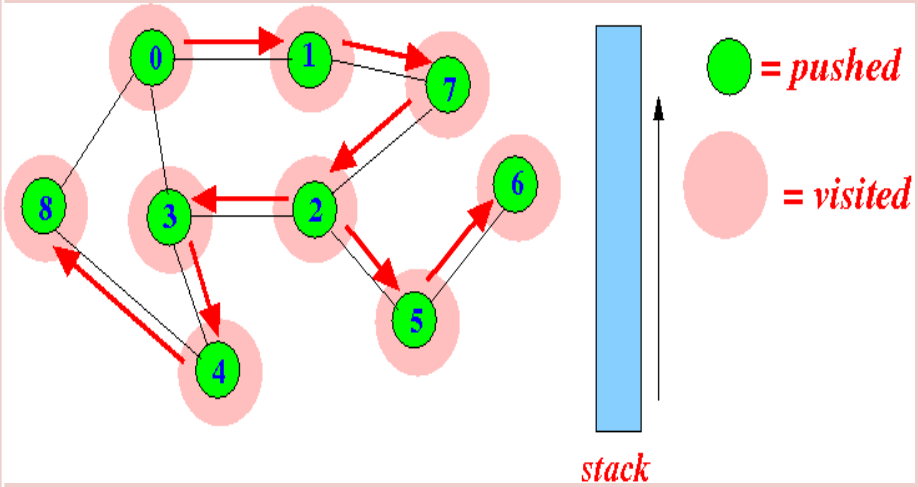
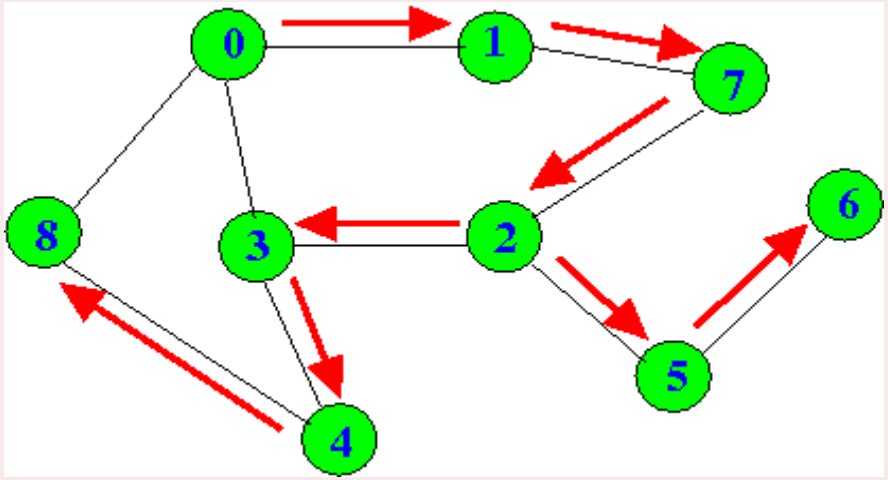
Step	Traversal	Description
5		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find C to be on the top of the stack.
6		Only unvisited adjacent node is from E is **C** now. So we visit **C**, mark it as visited and put it onto the stack.
7	DFS Traversal = A B C D E F	

Example of the DFS algorithm

Step	Traversal	Description
1		
2		**Initial state: node 0** is *pushed*
3		**State *after* visiting 0** **Push** the ***unvisited*** **neighbor nodes**: **8, 3, 1** (I used the reverse order to visit smaller node id first) **Next, visit** the ***top*** **node** in the **stack**: **1**
4		**State *after* visiting 1** **Push** the ***unvisited*** **neighbor nodes**: **7** **Next, visit** the ***top*** **node** in the **stack**: **7**

Step	Traversal	Description
5		**State *after* visiting 7** **Push** the ***unvisited* neighbor nodes**: 2 **Next, visit** the ***top* node**in the **stack**: 2
6		**State *after* visiting 2** **Push** the ***unvisited* neighbor nodes 5, 3** (Note: 3 is pushed again, and the previous value will be cancelled later -- as we will see) **Next, visit** the ***top* node**in the **stack**: 3
7		**State *after* visiting 3** **Push** the ***unvisited* neighbor nodes**: 4 **Next, visit** the ***top* node**in the **stack**: 4
8		**State *after* visiting 4** **Push** the ***unvisited* neighbor nodes**: 8 (Note: 8 is pushed again, and the previous value will be cancelled later -- as we will see) **Next, visit** the ***top* node**in the **stack**: 8

Step	Traversal	Description
9		**State *after* visiting 8** **Push** the ***unvisited* neighbor nodes: none** **Next, visit** the ***top* node** in the **stack: 5**
10		**State *after* visiting 5** **Push** the ***unvisited* neighbor nodes: 6** **Next, visit** the ***top* node** in the **stack: 6**
11		**State *after* visiting 6** **Push** the ***unvisited* neighbor nodes: none** **Next, visit** the ***top* node** in the **stack: 3** **3 is visited : skip**
12		**Next, visit** the ***top* node** in the **stack: 8** **8 is visited : skip**

Step	Traversal	Description
13		**DONE** (The **stack** hasbecome **empty**)
14		**Traversal order**